MATHEMATICS 14

EXAMPLES OF STUDENTS' RESPONSES

CLASSROOM
ASSESSMENT
MATERIALS

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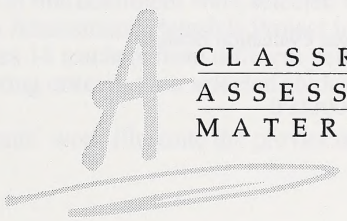

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Introduction

Purpose

The purpose of this document is to provide teachers, students, parents, and administrators with examples of students' responses that illustrate the provincial standards expected of students who complete Mathematics 14.

Contents

For each of the components, the *Examples of Students' Responses* document contains

- the student task
- the scoring criteria to be used by teachers to evaluate their students' work (these criteria can also be found in the *Teacher Manual*)
- examples of students' responses at each criteria "level"
- commentaries that illustrate and explain how the scoring criteria fit each response.

Each student response in this document is reproduced as it appeared in the assessment; that is, in the students' own handwriting.

Selection of Examples

The students' responses in this document were selected from those produced during the pilot testing of the Classroom Assessment Materials Project in May and June of 1996. A committee composed of Mathematics 14 teachers from different parts of the province reviewed and validated the assessments and scoring criteria, then selected students' responses.

These examples of students' work illustrate the provincial standards for students who complete Mathematics 14.

Considerations

Please note that

- the examples presented illustrate specific standards (scoring criteria), but are not necessarily typical of the responses submitted.
- the selected responses represent only a few of the possible approaches to each task. None of the examples is intended to serve as a model of a particular approach.
- you should consider each student example in light of the constraints of the assessment situation. Under assessment conditions, most students are able to prepare responses that must be considered as first draft only.

Contents

In addition to this Examples of Student's Responses, the Mathematics 14 Classroom Assessement Materials includes a complete set of Student Materials and a Teacher Manual in separate booklets.

Mathematics 14

End-of-Course Exam

- ***Written Response 1***
- ***Written Response 2***
- ***Written Response 3***

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Written Response 1: Student Task and Solution

(5 marks) **1.** A piece of photocopying paper is very thin, and its thickness cannot be measured directly. A 500-sheet package of photocopying paper measures 28 cm long × 22 cm wide × 5.1 cm thick. A 200-sheet box of facial tissues measures 22 cm long × 12 cm wide × 7.2 cm thick.

a. Calculate the thickness of one sheet of photocopying paper.

Solution

1 sheet is (5.1 cm)/500, or **0.0102 cm thick**

b. Calculate the thickness of one facial tissue, and explain why this calculated answer is likely to be very inaccurate.

Solution

Just taking the thickness of 200 sheets, 7.2 cm, and dividing it by 200 gives a **thickness of 0.036 cm** that is 3.5 times as thick as regular photocopying bond paper.

This does not allow for the air pockets and the double folding found in the loose packing of tissues in their boxes.

Task-Specific Scoring Criteria for Written Response 1

Scale score	Criteria
5	Complete answer to both parts, with supporting explanation shown. Final answers are correct, including units, and the communication is readily understandable.
4	**Either** final answers may be incorrect, and the communication may lack some clarity; **or** final numerical answers to both parts are correct, but either the explanations are inadequately supported, or imprecisely expressed, or the units are omitted.
3	**Either** a complete answer that has one major error **or** many minor errors present, but that indicates the calculation strategy, the approximate value of the thickness, and the written explanations in support; **or** the final numerical answers to both parts are correct, but they are poorly supported by written explanations, with many gaps needed to be filled in by the reader.
2	**Either** a reasonable value for the thickness in either part with some supporting detail; **or** a correct numerical answer to each part, with no supporting explanation.
1	A significant start made to the solution of the problem. Examples of significant starts include, but are not limited to, attempts at explaining the calculation strategy used, or awareness of the differences between bond paper and tissue, or a calculation of the thickness that involves a division.
0	**Either** off-topic; **or** an unreasonable estimate in part a., with no supporting detail; **or** a blank paper.

Written Response 1: Student Responses

This response would receive a score of **5**

Scoring Criteria

- Complete answer to both parts, with supporting explanations shown. Final answers are correct, including units, and the communication is readily understandable.

(5 marks) **1.** A piece of photocopying paper is very thin, and its thickness cannot be measured directly. A 500-sheet package of photocopying paper measures 28 cm long × 22 cm wide × 5.1 cm thick. A 200-sheet box of Kleenex tissues measures 22 cm long × 12 cm wide × 7.2 cm thick.

a. Calculate the thickness of 1 sheet of photocopying paper.

$\frac{5.1}{500} \quad \frac{n}{1}$

$n = 0.0102 \text{ cm}$

b. Calculate the thickness of 1 Kleenex tissue, and explain why this calculated answer is likely to be very inaccurate. Because the Kleenex is usually 2 ply.

$\frac{7.2}{200} \quad \frac{n}{1}$

$n = 0.036 \text{ cm}$

Commentary

This response receives a score of 5 because the student

- sets up a proportion involving the division of the thickness by 500 for photocopying paper, and by 200 for facial tissues
- performs the divisions correctly and includes the appropriate units in the answer
- provides a reasonable explanation for the facial tissue result by stating that facial tissue is 2-ply in texture

This response would receive a score of **4**

Scoring Criteria

- Final numerical answers to both parts are correct, but either the explanations are inadequately supported, or imprecisely expressed, or the units are omitted.

(5 marks) **1.** A piece of photocopying paper is very thin, and its thickness cannot be measured directly. A 500-sheet package of photocopying paper measures 28 cm long × 22 cm wide × 5.1 cm thick. A 200-sheet box of Kleenex tissues measures 22 cm long × 12 cm wide × 7.2 cm thick.

a. Calculate the thickness of 1 sheet of photocopying paper.

0.0102cm 5.1 ÷ 500 = 0.0102cm

b. Calculate the thickness of 1 Kleenex tissue, and explain why this calculated answer is likely to be very inaccurate.

0.036cm 7.2 ÷ 200
it is likly to be wrong because Kleenex tends to be puffer therefore it it likely to be wrong.

Commentary

This response receives a score of 4 because the student

- employs a strategy involving the division of the thickness by 500 for photocopying paper, and by 200 for facial tissues
- performs the divisions correctly and includes the appropriate units in the answer
- is somewhat unclear with the wording "because Kleenex tends to be puffer therefore it is likely to be wrong." Here the word "puffer" is probably intended to be the word "puffier," but the communication still is sufficiently imprecise to assign a score of 4, rather than 5. For a score of 5, some reference to "2-ply" or "pockets of air between the folds" would be needed

Responses without units of measure, but otherwise correct and including a clear explanation, would also receive a score of 4.

This response would receive a score of **3**

Scoring Criteria

- Final numerical answers to both parts are correct, but they are poorly supported by written explanations, with many gaps to be filled in by the reader.

(5 marks) **1.** A piece of photocopying paper is very thin, and its thickness cannot be measured directly. A 500-sheet package of photocopying paper measures 28 cm long × 22 cm wide × 5.1 cm thick. A 200-sheet box of Kleenex tissues measures 22 cm long × 12 cm wide × 7.2 cm thick.

a. Calculate the thickness of 1 sheet of photocopying paper.

$5.1 \div \simeq 0.0102$

b. Calculate the thickness of 1 Kleenex tissue, and explain why this calculated answer is likely to be very inaccurate.

$7.2 \div \simeq 0.036$ is wrong because Kleenex tissue is fluffy

Commentary

This response receives a score of 3 because the student

- obtains correct numerical answers of 0.0102 for photocopying paper, and 0.036 for facial tissues
- includes an explanation "because Kleenex tissue is fluffy," which, though colloquial, is essentially correct, even though somewhat imprecise
- leaves many gaps to be filled in by the reader, notably the divisors 500 in the first part and 200 in the second, as well as the units of measure, centimetres, in both parts

Other responses with a score of 3 have numerical errors in the thicknesses, but, compared to the response shown, provide more detailed support for the numerical answer.

This response would receive a score of **2**

Scoring Criteria

- A reasonable value for the thickness in either part with some supporting detail.

(5 marks) **1.** A piece of photocopying paper is very thin, and its thickness cannot be measured directly. A 500-sheet package of photocopying paper measures 28 cm long × 22 cm wide × 5.1 cm thick. A 200-sheet box of Kleenex tissues measures 22 cm long × 12 cm wide × 7.2 cm thick.

a. Calculate the thickness of 1 sheet of photocopying paper.

28 ÷500 .056

22 ÷500 .044

5.1 ÷500 .0102

.056 x .044 x .0102 per sheet of bond paper
I divided each by 500

b. Calculate the thickness of 1 Kleenex tissue, and explain why this calculated answer is likely to be very inaccurate.

Commentary

This response receives a score of 2 because the student

- recognizes the necessity of dividing by 500 to obtain the thickness of photocopying paper, and includes an explanation as part of the response
- carries out the numerical calculation correctly, but omits the centimetres as units of measure
- does not distinguish which of the three numbers, 0.056, 0.044 and 0.0102, represents the thickness of the paper
- leaves the second part blank

This response would receive a score of **1**

Scoring Criteria

- A significant start made to the solution of the problem. Examples of significant starts include, but are not limited to, attempts at explaining the calculation strategy used, or awareness of the differences between photocopying paper and tissue, or a calculation of the thickness that involves a division.

(5 marks) **1.** A piece of photocopying paper is very thin, and its thickness cannot be measured directly. A 500-sheet package of photocopying paper measures 28 cm long × 22 cm wide × 5.1 cm thick. A 200-sheet box of Kleenex tissues measures 22 cm long × 12 cm wide × 7.2 cm thick.

a. Calculate the thickness of 1 sheet of photocopying paper.

$$\frac{28\text{ cm} \times 22\text{ cm} \times 5.1\text{ cm}}{500}$$

$$= \frac{3141.6}{500}$$

$$= 6.28\text{ cm}$$

b. Calculate the thickness of 1 Kleenex tissue, and explain why this calculated answer is likely to be very inaccurate.

$$\frac{22\text{ cm} \times 12\text{ cm} \times 7.2\text{ cm}}{200}$$

$$= \frac{1900.8}{200}$$

$$= 9.5\text{ cm}$$

It's likely to be wrong because it's to dificult to measure kleenex tissue and the out come will not allways be the same

Commentary

This response receives a score of 1 because the student

- divided something by 500 in the first part, and by 200 in the second part
- incorrectly used the volume of the package of photocopying paper, and of the box of tissues, as a starting point, rather than the thickness of the package, or the thickness of the box of tissues
- obtained final answers of 6.28 cm for photocopying paper and 9.5 cm for tissue that are both unreasonably large

Even if the student had completed only one of the two divisions, the start would still have been enough for a score of 1. On the other hand, the result of the division by 500 or 200, depending on the part, has to result in a reasonable answer for a score of 2.

Written Response 2: Student Task and Solution

(5 marks) **2.** The following chart appeared in a weekend edition of a local newspaper. The chart indicates the favorite sorts of music of people who read three different newspapers from three different geographic areas.

Music Choices of 1995			
Type of Music	**Edmonton Journal**	**Calgary Herald**	**The Globe and Mail**
Rock and Roll	12	15	20
Western	13	55	15
Alternative	27	4	11
Rap	11	2	2
Classical	0	8	27
Reggae	26	3	5
Pop	6	7	4
Heavy Metal	5	6	6
Total	100	100	90

a. How does the chart show that readers of each of the three newspapers have **different** musical tastes?

Solution

Readers of the **Edmonton Journal** like Alternative and Reggae more than do readers of the other two papers (27% and 26%, compared with 4% and 3%, or with 12% and 5%).

Readers of the **Calgary Herald** like Western more than do readers of the other two papers (55% compared with 13% and 7%).

Readers of the **Globe and Mail** like Classical more than do readers of the other two papers (30% compared with 0% and 8%).

The tastes in Rock and Roll, Pop, Heavy Metal, and possibly Rap are roughly the same in the three centres.

b. Prepare a circle graph (pie chart) showing the music preferences of the 90 Globe and Mail readers. Show calculations of sector angles in the space provided, and use the circle below for your completed work.

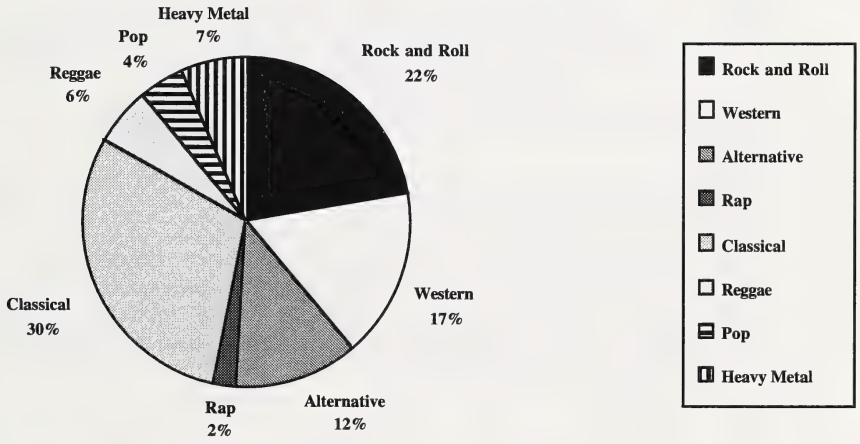
Solution

The Globe and Mail sample is 90, so each member of the sample corresponds to $\frac{360}{90}$, or 4.0°.

So Rock and Roll corresponds to 80°, Western to 60°, Alternative to 44°, Rap to 8°, Classical to 108°, Reggae to 20°, Pop to 16°, and Heavy Metal to 24°.

The circle graph is shown below.

Music Preferences of 90 Globe and Mail Readers



Task-Specific Scoring Criteria

Scale score	Criteria
5	Complete and correct answer to both parts, with supporting detail shown. Final answers are correct, the conclusions are strictly supported by the data, and the communication is readily understandable.
4	Complete answer to both parts, with supporting detail shown. Final answers are correct, the conclusions are reasonably consistent with the data, and the communication is generally understandable. Minor errors may be present in the details.
3	**Either** a complete answer that has one major error **or** many minor errors present, but that indicates the reading of the data table, the calculation of the sector angles, the drawing of the pie graph, and the written explanations in support; **or** a complete and correct pie graph; **or** a set of conclusions that are supported by the data, with little significant attempt at a pie graph.
2	**Either** an answer that has major errors present, but that indicates some reading of the data table, an attempt at the calculation of the sector angles, a rough drawing of a pie graph, and some written explanations in support; **or** a complete pie graph, with minor errors in the calculation of the sector angles; **or** a set of conclusions that are approximately supported by the data, with no attempt at a pie graph.
1	A significant start made to the solution of the problem. Examples of significant starts include, but are not limited to, the calculation of one or more sector angles, or the construction of a pie graph which includes a sector for each category, or a summary paragraph that makes some reference to data that are in the chart or in the pie graph.
0	**Either** off-topic; **or** an unreasonable estimate in part a., with no supporting detail; **or** a blank paper.

Student responses to Written Response 2 follow on page 14

Student Responses

This response would receive a score of **5**

Scoring Criteria

- Complete and correct answer to both parts, with supporting detail shown. Final answers are correct, the conclusions are strictly supported by the data, and the communication is readily understandable.

(5 marks) **2.** The following chart appeared in a weekend edition of a local newspaper. The chart indicates the favourite sorts of music of people from three different geographic areas.

Music Choices of 1995

Type of music	Edmonton Journal	Calgary Herald	Globe and Mail
Rock and Roll	12	15	20
Western	13	55	15
Alternative	27	4	11
Rap	11	2	2
Classical	0	8	27
Reggae	26	3	5
Pop	6	7	4
Heavy Metal	5	6	6
Totals	100	100	90

a. Draw evidence from the chart that shows that readers of the three newspapers each have **different** musical tastes.

From the Edmonton Journal the favorite type of music is Alternativ for the Calgary Herald is western music and for Globe & Mail the favorite is classical

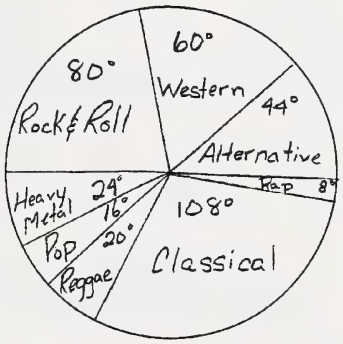
Papers	Music
Edmonton Journal	Alternative
Calgary Herald	Western
Globe & Mail	Classical

Continued

Continued

R&R - 360° x 20 ÷ 90 = 80°
W - 360° x 15 ÷ 90 = 60°
Alt. - 360° x 11 ÷ 90 = 44°
Rap - 360° x 2 ÷ 90 = 8°
Clas. - 360° x 27 ÷ 90 = 108°
Reggae - 360° x 5 ÷ 90 = 20°
Pop - 360° x 4 ÷ 90 = 16°
H.M - 360° x 6 ÷ 90 = 24°

b. Prepare a circle graph (pie chart) showing the music preferences of the 90 Globe and Mail readers. Show calculations of sector angles in the space provided, and use the circle below for your completed work.



Commentary

This response receives a score of 5 because the student

- is careful in the selection of appropriate data from the chart, rather than merely paraphrasing the data set
- uses the different types of music at the top of the popularity charts for the three newspapers as evidence that tastes differ. There is no need of backup data when the data quoted is so convincing
- uses the idea of ratio and proportion to show how each of the sector angles were calculated. It is clear that the student could have done the calculations mentally and just written down the answers. The showing of work is an attempt to convince the reader and to explain the solution process
- provides a circle graph that is either a construction or a very accurate sketch, and attempts to communicate the data even more effectively by recording the sector angles for each of the eight sectors of the circle graph. However, in this case, percentages would have served as better labels for the relevant sector sizes

This response would receive a score of **4**

Scoring Criteria

- Complete answer to both parts, with supporting detail shown. Final answers are correct, the conclusions are reasonably consistent with the data, and the communication is generally understandable. Minor errors may be present in the details.

(5 marks) **2.** The following chart appeared in a weekend edition of a local newspaper. The chart indicates the favourite sorts of music of people from three different geographic areas.

Music Choices of 1995

Type of music	Edmonton Journal	Calgary Herald	Globe and Mail
Rock and Roll	12	15	20
Western	13	55	15
Alternative	27	4	11
Rap	11	2	2
Classical	0	8	27
Reggae	26	3	5
Pop	6	7	4
Heavy Metal	5	6	6
Totals	100	100	90

a. Draw evidence from the chart that shows that readers of the three newspapers each have **different** musical tastes.

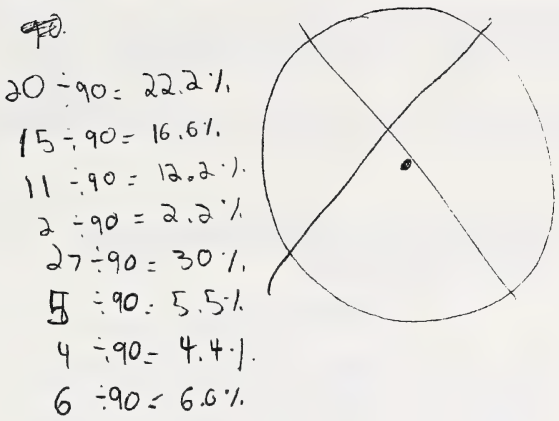
Edmonton - 30% like Alternative
and 28.8% like Raggae

Calgary - 61.1% like Western

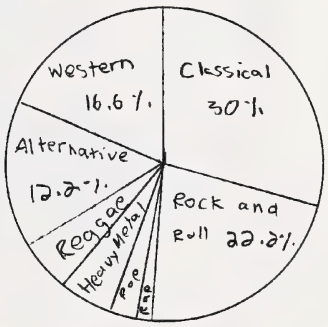
Globe + Mail - 30% like Classical

Continued

Continued



b. Prepare a circle graph (pie chart) showing the music preferences of the 90 Globe and Mail readers. Show calculations of sector angles in the space provided, and use the circle below for your completed work.



Commentary

This response receives a score of 4 because the student

- is careful in the selection of appropriate data from the chart, rather than merely paraphrasing the data set
- uses the different types of music at the top of the popularity charts for the three newspapers as evidence that tastes differ. In the response, there was an assumption that the totals were 90 for all three papers, rather than 100 for the Edmonton and Calgary papers and 90 for the Toronto sample. This miscalculation is minor, and does not affect the validity of the conclusion
- uses percentages to determine the sector angles for the circle graph, but omits the details of how the percentages were converted to degrees for the circle graph. It is not obvious whether the sector angles were sketched or drawn with a protractor, but the sizes are very close to the accurate sizes. The percentage labels are appropriate for the graph

This response would receive a score of **3**

Scoring Criteria

- a set of conclusions that are supported by the data, with little significant attempt at a pie graph.

(5 marks) **2.** The following chart appeared in a weekend edition of a local newspaper. The chart indicates the favourite sorts of music of people from three different geographic areas.

Music Choices of 1995

Type of music	Edmonton Journal	Calgary Herald	Globe and Mail
Rock and Roll	12	15	20
Western	13	55	15
Alternative	27	4	11
Rap	11	2	2
Classical	0	8	27
Reggae	26	3	5
Pop	6	7	4
Heavy Metal	5	6	6
Totals	100	100	90

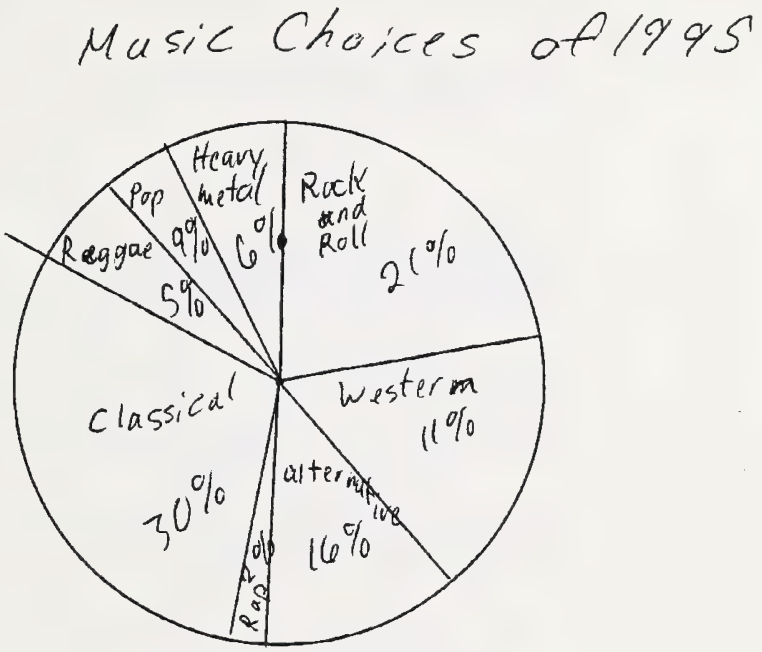
a. Draw evidence from the chart that shows that readers of the three newspapers each have **different** musical tastes.

In the Edmonton Journal the highest was Alternative, in the Calgary Herald Western was the highest, and Classical was highest in Globe and Mail. From the information gathered it is evident that different readers of news papers liked different types of music.

Continued

Continued

b. **Prepare a circle graph (pie chart) showing the music preferences of the 90 Globe and Mail readers. Show calculations of sector angles in the space provided, and use the circle below for your completed work.**



Commentary

This response receives a score of 3 because the student

- is careful in the selection of appropriate data from the chart, rather than merely paraphrasing the data set
- uses the different types of music at the top of the popularity charts for the three newspapers as evidence that tastes differ. There is no need of backup data when the data quoted is so convincing
- makes an attempt at a circle graph, but the percentages are sometimes correct (the 30% for Classical), and sometimes incorrect (the 11% for Western, instead of the correct 17%). Moreover, the sketched sector angles are not even consistent with the miscalculated sector angles

This response would receive a score of **2**

Scoring Criteria

- an answer that has major errors present, but that indicates some reading of the data table, an attempt at the calculation of the sector angles, a rough drawing of a pie graph, and some written explanations in support.

(5 marks) **2.** The following chart appeared in a weekend edition of a local newspaper. The chart indicates the favourite sorts of music of people from three different geographic areas.

Music Choices of 1995

Type of music	Edmonton Journal	Calgary Herald	Globe and Mail
Rock and Roll	12	15	20
Western	13	55	15
Alternative	27	4	11
Rap	11	2	2
Classical	0	8	27
Reggae	26	3	5
Pop	6	7	4
Heavy Metal	5	6	6
Totals	100	100	90

a. Draw evidence from the chart that shows that readers of the three newspapers each have **different** musical tastes.

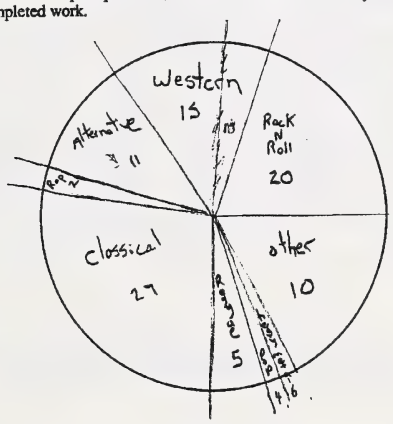
The readers all have diffrt taste in musi

Continued

Continued

$20 \times 3.6 = 72°$
$13 \times 3.6 = 54°$
$11 \times 3.6 = 39.6°$
$2 \times 3.6 = 7.2°$
$27 \times 3.6 = 97.2$
$5 \times 3.6 = 18°$
$4 \times 3.6 = 18.4°$
$6 \times 3.6 = 21.6°$

b. Prepare a circle graph (pie chart) showing the music preferences of the 90 Globe and Mail readers. Show calculations of sector angles in the space provided, and use the circle below for your completed work.



Commentary

This response receives a score of 2 because the student

- calculates the sector angles by multiplying totals by 3.6 degrees instead of 4, thereby assuming a sample of 100 instead of 90 for the Globe and Mail readers. The 10% that was unassigned is allocated as "other" in the bottom right part of the circle as a fudging factor
- draws a sketch circle graph instead of using a protractor, which is reasonable given the time constraints inherent in the exam situation. The relative sizes of the eight sectors are reasonably close to being accurate
- does not go beyond a restatement of the question in trying to extract information from the chart

This response would receive a score of **1**

Scoring Criteria

- A significant start made to the solution of the problem. Examples of significant starts include, but are not limited to, the calculation of one or more sector angles, or the construction of a pie graph which includes a sector for each category, or a summary paragraph that makes some reference to data that are in the chart or in the pie graph.

(5 marks) **2.** The following chart appeared in a weekend edition of a local newspaper. The chart indicates the favourite sorts of music of people from three different geographic areas.

Music Choices of 1995

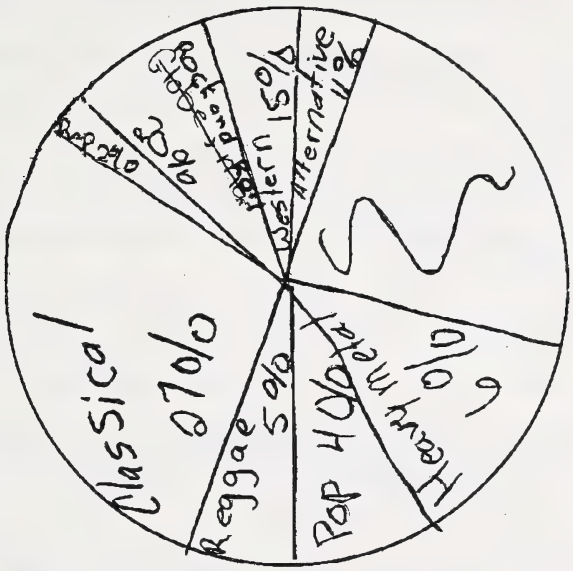
Type of music	Edmonton Journal	Calgary Herald	Globe and Mail
Rock and Roll	12	15	20
Western	13	55	15
Alternative	27	4	11
Rap	11	2	2
Classical	0	8	27
Reggae	26	3	5
Pop	6	7	4
Heavy Metal	5	6	6
Totals	100	100	90

a. Draw evidence from the chart that shows that readers of the three newspapers each have **different** musical tastes.

Continued

Continued

b. **Prepare a circle graph (pie chart) showing the music preferences of the 90 Globe and Mail readers. Show calculations of sector angles in the space provided, and use the circle below for your completed work.**



Commentary

This response receives a score of 1 because the student

- makes a significant start by identifying the nature of a circle graph and the use of sector angles to show the sizes of the different categories
- makes a very rough circle graph with a large unassigned section in the top right. Some of the sector angles, such as the Classical at 27%, are reasonably accurate, but others, such as the Heavy Metal at 6% and Western at 15%, are very much out of proportion
- makes an illegible entry for the chart interpretations, and then decides to cross it out

Written Response 3: Student Task and Solution

(5 marks) **3.** Woody works in a hamburger restaurant. The restaurant uses toothpicks to hold the hamburgers together. Woody's boss found an excellent buy on a million toothpicks, and asked Woody to pick them up. This sounded like a lot of toothpicks. Woody wondered if a packing case containing these toothpicks would fit into the trunk of his small car.

a. One box holds 250 toothpicks. A carton holds 20 boxes. How many toothpicks does this carton hold?

Solution

(20 boxes/carton)(250 toothpicks/box)**= 5 000 toothpicks**

b. How many cartons are needed to build a stack containing one million toothpicks?

Solution

1 million toothpicks is (1 000 000/5 000) or **200 cartons**

c. One box of toothpicks measures 5.2 cm long × 2.7 cm wide × 1.3 cm high, and a carton of 20 boxes measures 10.4 cm long × 13.5 cm wide × 2.6 cm high.

What could be the dimensions of a packing case that holds a million toothpicks?

The trunk of Woody's car measures 0.7 m long × 1.3 m wide × 0.4 m high. Will this packing case fit into the car trunk?

Solution 1 using cartons

200 cartons could be 10 long by 5 deep by 4 high
Dimensions would be (10 × 10.4 cm) × (5 × 13.5 cm) × (4 × 2.6 cm)
This is 1.04 m long by 0.675 m deep by 10.4 cm high, and this fits the car trunk easily

Solution 2 using boxes

4 000 boxes could be 25 long by 20 deep by 8 high
Dimensions would be (25 × 5.2 cm) × (20 × 2.7 cm) × (8 × 1.3 cm)
This is 1.3 m long by 0.54 m deep by 10.4 cm high, and this fits the car trunk easily

Note that these solutions are not unique, and that there are many ways of fitting one million toothpicks into the car trunk.

Mathematics 14

Task-Specific Scoring Criteria

Scale score	Criteria
5	Complete answers to all parts, with supporting detail shown. Final answers are correct, and the communication is readily understandable.
4	**Either** a complete and numerically correct answer to all three parts, with supporting detail shown, and having a packing case that holds a million toothpicks, but with little or no attempt to fit the case into the car trunk; **or** a complete answer to all three parts, with supporting detail shown, having a packing case that holds at least a million toothpicks, and which fits in the car trunk. Any calculation errors present still require the design of a packing case that fits into a car trunk, and that holds the calculated number of toothpicks.
3	**Either** a complete answer that has one major error **or** many minor errors present, but that indicates the calculation of the dimensions of a carton, the number of boxes needed to hold a million toothpicks, and the dimensions of the packing case; **or** a correct answer to one or two parts, with supporting detail shown and making an attempt either to find the volume or dimensions of a packing case that holds a million toothpicks, or a packing case that fits in the car trunk.
2	**Either** two correct numerical answers, with or without support; **or** reasonable answers to the first two parts, together with an attempt to find the volume or the dimensions of either the carton or the packing case.
1	**Either** one correct numerical answer, with or without support; **or** a significant start made to the solution of the problem. Examples of significant starts include, but are not limited to, a calculation of the number of toothpicks in a carton, or an attempt to find the number of boxes or the number of cartons needed to hold a million toothpicks.
0	**Either** off-topic; **or** an unreasonable answer in part a. or part b., with no supporting detail; **or** a blank paper.

Written Response 3: Student Responses

This response would receive a score of **5**

Scoring Criteria

- Complete answers to all parts, with supporting detail shown. Final answers are correct, and the communication is readily understandable.

(5 marks) **3.** Woody works in a hamburger restaurant. The restaurant uses toothpicks to hold the hamburgers together. Woody's boss found an excellent buy on a million toothpicks, and asked Woody to pick them up. This sounded like a lot of toothpicks. Woody wondered if a packing case containing these toothpicks would fit into the trunk of his small car.

a. One box holds 250 toothpicks. A carton holds 20 boxes. How many toothpicks does this carton hold?

box x 250 toothpicks
Carton 20 x 250 = 5000 picks

b. How many cartons are needed to build a stack that can hold one million toothpicks?

1000 000 ÷ 250 = 4000
4000 are needed to build a stack

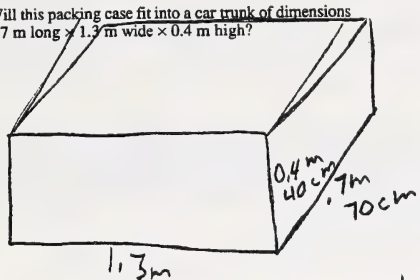
Continued

Continued

c. One box of toothpicks measures 5.2 cm long × 2.7 cm wide × 1.3 cm high and a carton of 20 boxes measures 10.4 cm long × 13.5 cm wide × 2.6 cm high.

What could be the dimensions of a packing case that holds a million toothpicks?

Will this packing case fit into a car trunk of dimensions 0.7 m long × 1.3 m wide × 0.4 m high?

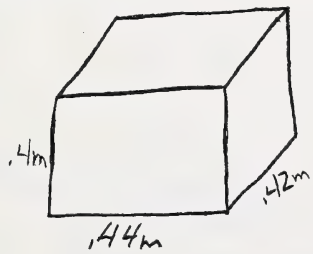


box 250 364000cm3

carton 20

1 carton box of million picks

carton - 200= 10.4 x 13.5 x 2.6 = 365.04cm = 73008cm3



.4 m
.44 m
.42m

.4m x .44m x .42m equal 1 box of million picks

Commentary

This response receives a score of 5 because the student

- correctly calculates the number of toothpicks in a carton, and cartons in a packing case. The 200 cartons in the packing case is recorded as part of the response to part c., rather than as a response to part b. The 4 000 given in part b. refers to the number of boxes in a packing case. This wrong answer in part b. is corrected in part c. and so no marks are deducted
- uses the dimensions of a carton to find the volume of the carton and the volume of the 200 cartons that make up the packing case
- uses trial and error to find the volume of a packing case with volume equal to 73 000 cm^3, and with dimensions that fit into the trunk of Woody's car
- uses a height of 0.4 m and a diagram to explain how the trial dimensions for the packing case all fit into Woody's car

There were no responses in the sample that would have received a score of **4**

Scoring Criteria

- **Either** a complete and numerically correct answer to all three parts, with supporting detail shown, and having a packing case that holds a million toothpicks, but with little or no attempt to fit the case into the car trunk;
- **or** a complete answer to all three parts, with supporting detail shown, having a packing case that holds at least a million toothpicks, and which fits in the car trunk. Any calculation errors present still require the design of a packing case that fits into a car trunk, and that holds the calculated number of toothpicks.

Commentary

Responses that would receive a score of 4 fall into two types. The first type of response:

- correctly calculates the number of toothpicks in a carton, and cartons in a packing case
- uses the dimensions of a carton to find the volume of the carton, and the volume of the 200 cartons that make up the packing case
- makes no attempt to link the volume of the packing case to the dimensions of the car trunk or to the dimensions of a possible packing case

These responses look like a shortened version of the response that received a score of 5.

The second type of response:

- correctly calculates the number of toothpicks in a carton, and cartons in a packing case
- uses the dimensions of a carton to find the volume of the carton, and the volume of the 200 cartons that make up the packing case. There may be minor calculation errors present
- makes an attempt to link the volume of the packing case to the dimensions of the car trunk or to the dimensions of a possible packing case that will fit into the car trunk

This response would receive a score of **3**

Scoring Criteria

- a correct answer to one or two parts, with supporting detail shown, making an attempt either to find the volume or dimensions of a packing case that holds a million toothpicks, or a packing case that fits in the car trunk.

(5 marks) **3.** Woody works in a hamburger restaurant. The restaurant uses toothpicks to hold the hamburgers together. Woody's boss found an excellent buy on a million toothpicks, and asked Woody to pick them up. This sounded like a lot of toothpicks. Woody wondered if a packing case containing these toothpicks would fit into the trunk of his small car.

18.25 = 250

a. One box holds 250 toothpicks. A carton holds 20 boxes. How many toothpicks does this carton hold?

250 x 20 = 5000 tooth picks

b. How many cartons are needed to build a stack that can hold one million toothpicks?

You would Need 4000 boxes or 200 cartons

c. One box of toothpicks measures 5.2 cm long × 2.7 cm wide × 1.3 cm high and a carton of 20 boxes measures 10.4 cm long × 13.5 cm wide × 2.6 cm high.

What could be the dimensions of a packing case that holds a million toothpicks?

Will this packing case fit into a car trunk of dimensions 0.7 m long × 1.3 m wide × 0.4 m high?

1.2 m x 0.6 m x 0.4 m = 28.8 m²

Commentary

This response receives a score of 3 because the student

- correctly calculates the number of toothpicks in a carton and the number of cartons needed to build the packing case for one million toothpicks
- calculates, with a minor slip of the decimal, the volume of something that will fit into the trunk of the car, but does not relate this volume to the volume of a box or the volume of a carton. If this approach had been continued with the correct volume of 0.288 m^3, this crate would be able to hold 760 cartons, more than enough to accommodate the million toothpicks

This response would receive a score of **2**

Scoring Criteria

- reasonable answers to the first two parts, together with an attempt to find the volume or the dimensions of either the carton or the packing case.

(5 marks) 3. Woody works in a hamburger restaurant. The restaurant uses toothpicks to hold the hamburgers together. Woody's boss found an excellent buy on a million toothpicks, and asked Woody to pick them up. This sounded like a lot of toothpicks. Woody wondered if a packing case containing these toothpicks would fit into the trunk of his small car.

a. One box holds 250 toothpicks. A carton holds 20 boxes. How many toothpicks does this carton hold?

250 in 1 box
250 x 20 boxes = 5000 toothpicks

b. How many cartons are needed to build a stack that can hold one million toothpicks?

5000+5000 = 10 000
40 boxes

40 boxes holds 5000 and thats half of the toothpicks the total boxes for 1 Million toothpicks must be 40 boxes

Continued

Continued

c. One box of toothpicks measures 5.2 cm long × 2.7 cm wide × 1.3 cm high and a carton of 20 boxes measures 10.4 cm long × 13.5 cm wide × 2.6 cm high.

What could be the dimensions of a packing case that holds a million toothpicks?

Will this packing case fit into a car trunk of dimensions 0.7 m long × 1.3 m wide × 0.4 m high?

put them in piles of 4 there would be 10 piles in the trunk

Commentary

This response receives a score of 2 because the student

- calculates the number of toothpicks in a carton correctly in the first part
- confuses 1 000 000 with 10 000 and concludes that 40 boxes, rather than 4 000 boxes, are needed for the million toothpicks
- claims that the 40 boxes fit in the trunk easily by piling them as 10 piles of 4 boxes each

The more common response with a score of 2 is a response with correct answers, supported or unsupported, to the first two parts, and little or nothing for the third part.

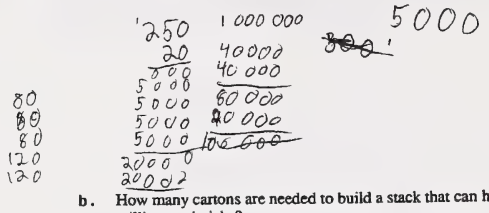
This response would receive a score of **1**

Scoring Criteria

- a significant start made to solving the problem. Examples of significant starts include, but are not limited to, a calculation of the number of toothpicks in a carton, or an attempt to find the number of boxes or the number of cartons needed to hold a million toothpicks.

(5 marks) **3.** Woody works in a hamburger restaurant. The restaurant uses toothpicks to hold the hamburgers together. Woody's boss found an excellent buy on a million toothpicks, and asked Woody to pick them up. This sounded like a lot of toothpicks. Woody wondered if a packing case containing these toothpicks would fit into the trunk of his small car.

a. One box holds 250 toothpicks. A carton holds 20 boxes. How many toothpicks does this carton hold?



b. How many cartons are needed to build a stack that can hold one million toothpicks?

300 Boxes

c. One box of toothpicks measures 5.2 cm long × 2.7 cm wide × 1.3 cm high and a carton of 20 boxes measures 10.4 cm long × 13.5 cm wide × 2.6 cm high.

What could be the dimensions of a packing case that holds a million toothpicks?

Will this packing case fit into a car trunk of dimensions 0.7 m long × 1.3 m wide × 0.4 m high?

Commentary

This response receives a score of 1 because the student

- calculates the number of toothpicks in a carton correctly in the first part
- gives an unsupported incorrect answer of 300 boxes in the second part, even though the problem asked for cartons
- leaves the third part blank

Even if the supporting arithmetic in the first part had been omitted, the response would still have been scored as 1, since an unsupported correct answer is scored as 1 according to the *General Scoring Criteria*.

Performance Assessment

- *Task 1: Objects in a Measuring Cylinder*
- *Task 2: Determining the Area of a Quadrilateral*
- *Task 3: Does One 12-Sided Die Equal Two 6-Sided Dice?*

CLASSROOM
ASSESSMENT
MATERIALS

The discussion of Task 1 starts on page 36

Task 1: Objects in a Measuring Cylinder
Student Task and Solution

If solid objects are placed in a jar of water, the water level will rise. The more objects that are placed in the jar, the more the water level will rise. This investigation is designed to relate the rise in water level to the number of identical objects placed in the jar. After taking your measurements and drawing a graph, you have to make some predictions.

Student instructions and information

Part 1: **a.** Fill your measuring cylinder half full of water.

b. Measure the volume of the water to the nearest millilitre. Record the result in a chart.

c. Put one object in the measuring cylinder.

d. Measure the new volume of the water and record the result in the chart.

e. Repeats steps c. and d. several times.

Part 2: Complete your chart of measurements.

Part 3: Display your results in an appropriate graph. The graph should have measured volume on the vertical axis and the number of objects on the horizontal axis.

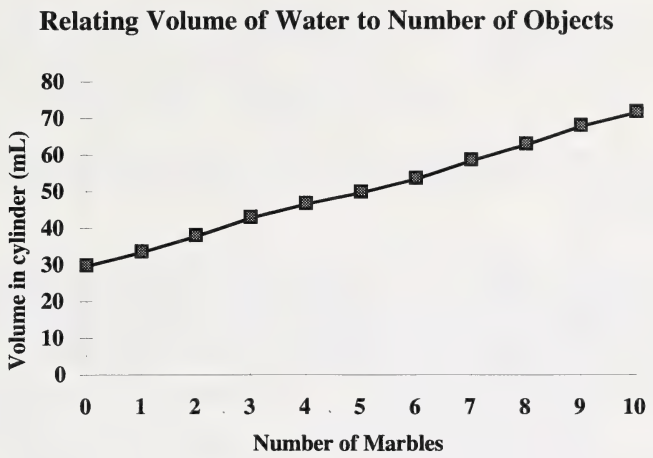
Chart of measurements

Solution

Number of Marbles	Volume of Water (mL)
0	30
1	34
2	38
3	42
4	47
5	51
6	55
7	58
8	62
9	66
10	70

Graph Of Water Volume And Number Of Objects

Solution



Predictions

Average volume of one object:

Solution

From the graph, 10 marbles occupy (72 – 30) mL, or 42 mL

So **volume of 1 marble = 4.2 mL**

Volume in the measuring cylinder if 20 objects have been put in the water:

Solution

30 mL + 20 × 4 mL = **110 mL by extrapolation**

Task-Specific Scoring Criteria

Mathematical Content Scale

Scale Score	Criteria
5	A complete and correct solution, with measurement table completed, a line or bar graph drawn, and predictions and conclusions made that follow from the data obtained.
4	**Either** the task is complete, with the measurements table completed, a line or bar graph (not necessarily best-fit) drawn, and predictions and conclusions drawn from the data. There may be minor omissions, inaccuracies, or errors in the details of the data treatment and the drawing of conclusions; **or** the task is correct but incomplete, with the measurements table completed, a line or bar graph (not necessarily best-fit) drawn, and either a reasonable prediction of the 20-marble volume or a reasonable conclusion for the volume of a single marble drawn from the measurement table or the graph.
3	**Either** the task is complete, with the measurements table completed, a line or bar graph drawn, and conclusions drawn from the data. There are significant omissions or errors in the details of the data treatment and the drawing of conclusions; **or** a completed, correct measurement table, together with some form of line or bar graph, but with little attempt to draw any systematic conclusions.
2	**Either** a completed, correct, measurement table; **or** an answer that shows an ability to follow the question directions, so as to produce a measurement table, a graph, and some conclusions. However, the number of errors present in the solution render all the data treatments unreliable.
1	A significant start made to the problem, such as (but not limited to) the setting up of a measurement table, together with one or two data points, or the drawing of a line graph, or a reasonable prediction of either the 20-marble volume or the volume of a single marble.
0	**Either** off-topic; **or** a blank paper.

Continued

Mathematics 14 – Continued

Communication Skills Scale

This scale is used **independently** of the mathematical content scale. A well-laid-out, systematic strategy with conclusions clearly stated can receive a scale score of 3, even if the comparison itself is based on data that either contain gross errors in the calculations of the supporting numbers, or reflect serious errors in measurement.

Scale Score	Criteria
3	A clear solution that includes a data table, shows the results of any computations in a systematic, ordered manner, and includes full concluding statements. The line or bar graph generally respects the normal conventions of axis labeling, graph drawing, and graph titling. Few, if any, gaps are left to be filled in by the reader.
2	A solution that includes a data table, shows the results of any computations in some manner, and includes some form of concluding statement. The line or bar graph makes an attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Some gaps are left to be filled in by the reader.
1	A solution that may provide a data table, shows the results of any computations in a somewhat disorganized manner, and may not even include any form of concluding statement. The line or bar graph makes little attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Large gaps are left to be filled in by the reader.
0	**Either** insufficient evidence of communication skills; **or** a blank paper.

Task 1: Student Responses

This response would receive a score of	**5**	*for mathematical content*
and would receive a score of	**3**	*for communication skills*

Scoring Criteria

Mathematical Content

- A complete and correct solution, with measurement table completed, a line or bar graph drawn, and predictions and conclusions made that follow from the data obtained.

Communication Skills

- A clear solution that includes a data table, shows the results of any computations in a systematic, ordered manner, and includes full concluding statements. The line or bar graph generally respects the normal conventions of axis labeling, graph drawing, and graph titling. Few, if any, gaps are left to be filled in by the reader.

Task 1: Marbles In a Measuring Cylinder

If solid objects are placed in a jar of water, the water level will rise. The more objects that are placed in the jar, the more the water level will rise. This investigation is designed to relate the rise in water level to the number of identical objects placed in the jar.

Student instructions and information

Part 1: **a.** Fill your measuring cylinder half full of water.

b. Measure the volume of the water to the nearest millilitre. Record the result in a chart.

c. Put one marble in the measuring cylinder.

d. Measure the new volume of the water and record the result in the chart.

e. Repeats steps c. and d. several times.

Part 2: Complete your chart of measurements.

Part 3: Display your results in an appropriate graph. The graph should have measured volume on the vertical axis and the number of marbles on the horizontal axis.

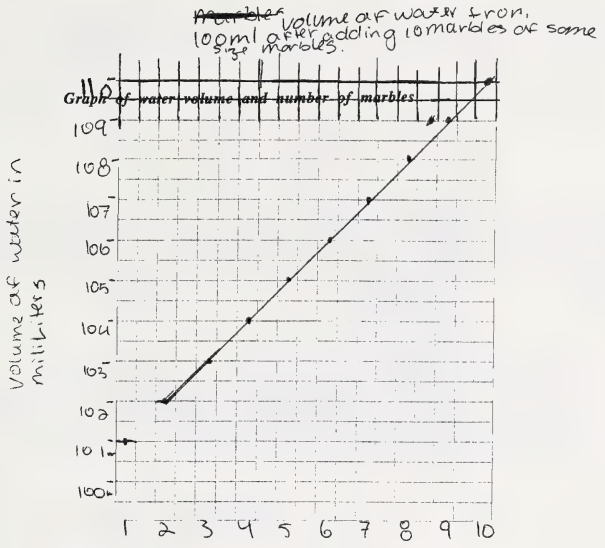
Chart of measurements

Volume of water to nearest millilitre

half Full or 100	1 marble	2	3	4	5	6	7	8	9	10
100 ml	100 ml	102 ml	103 ml	104 ml	105 ml	106 ml	107 ml	108 ml	109 ml	110 ml

Continued

Continued



Predictions

Average volume of one marble:

1 gram

Volume in the measuring cylinder if 20 marbles have been put in the water:

My prediction is that after adding 20 same sized marbles the volume would be 120 millilitres because as the chart or graph shows they equally went up by 1 measure so I assume with the same conditions this would continue

Commentary

*This response receives a score of 5 for **mathematical content** because the student*

- completes a table of measurements, and converts this table into the form of a line graph
- makes valid predictions, both for the volume after 20 marbles and for the average volume of one marble
- confuses the units for volume, stating the average value for one marble as 1 gram not 1 mL, but this error can be overlooked as an editorial error in an exam situation

*This response receives a score of 3 for **communication skills** because the student*

- provides a chart and a graph that are easy to read, and that generally are consistent with the conventions of mathematical and scientific communication. Particularly impressive is the inclusion of units, millilitres and marbles, on the axis labels
- provides a basis for the prediction for 20 marbles that is clear and makes the distinction between what is known and what is assumed, using the words "because as the chart or graph shows, they equally went up by 1 measure so I assume with the same conditions this would continue"

This response would receive a score of	**4**	*for mathematical content*
and would receive a score of	**2**	*for communication skills*

Scoring Criteria

Mathematical Content

- The task is complete, with the measurements table completed, a line or bar graph (not necessarily best-fit) drawn, and predictions and conclusions drawn from the data. There may be minor omissions, inaccuracies, or errors in the details of the data treatment and the drawing of conclusions.

Communication Skills

- A solution that includes a data table, shows the results of any computations in some manner, and includes some form of concluding statement. The line or bar graph makes an attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Some gaps are left to be filled in by the reader.

Task 1: Marbles In a Measuring Cylinder

If solid objects are placed in a jar of water, the water level will rise. The more objects that are placed in the jar, the more the water level will rise. This investigation is designed to relate the rise in water level to the number of identical objects placed in the jar.

Student instructions and information

Part 1: **a.** Fill your measuring cylinder half full of water.

b. Measure the volume of the water to the nearest millilitre. Record the result in a chart.

c. Put one marble in the measuring cylinder.

d. Measure the new volume of the water and record the result in the chart.

e. Repeats steps c. and d. several times.

Part 2: Complete your chart of measurements.

Part 3: Display your results in an appropriate graph. The graph should have measured volume on the vertical axis and the number of marbles on the horizontal axis.

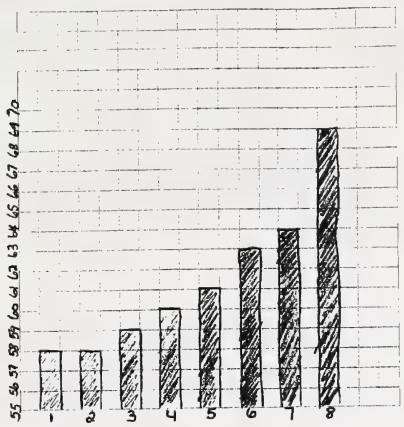
Chart of measurements

Starting 57 ml
1 marbles 58 ml
2 marbles 58 ml
3 marbles 59 ml
4 marbles 60 ml
5 marbles 61 ml
6 marbles 63 ml
7 marbles 64 ml
8 marbles 69 ml

Continued

Continued

Graph of water volume and number of marbles



Predictions

Average volume of one marble:

$V = 2\pi r$

$2 \times 3.14 \times 0.5 = 3.14$

Volume in the measuring cylinder if 20 marbles have been put in the water:

20 - 8 = 12
1 marble roughly raises water 1 mL
69 + 14 = 83 mL
I estimate it will raise 14 more mL to 83 mL

Commentary

This response receives a score of 4 for ***mathematical content*** *because the student*

- completes the chart of measurements with reasonable values for most of the volumes. The value quoted for 8 marbles may have been a misreading of the measuring cylinder or a clerical error in recording the value. Given the severely limited time that a student spends gathering data, this error is excusable
- converts the chart into an appropriate bar graph
- makes a reasonable, but not strictly accurate prediction for 20 marbles, adding on 14 mL for 12 marbles (rather more than double the 6 mL for the first 6 marbles) on to the 8 marble data point; and includes a statement "1 marble roughly raises water 1 mL" as an answer for the average volume of 1 marble. The volume calculation, $V = 2\pi r$, with the answer of 3.14 is ignored completely

This response receives a score of 2 for ***communication skills*** *because the student*

- communicates the problem solving strategy in a readily understandable form, with the conventions of mathematical communication being generally respected
- leaves some gaps in the communication, notably any explanation of the switch from 12 marbles to 14 mL the placing of the answer to the first prediction as part of the response to the second prediction, and the omission of the axis labels on the two axes

This response would receive a score of	**4**	*for mathematical content*
and would receive a score of	**1**	*for communication skills*

Scoring Criteria

Mathematical Content

- The task is complete, with the measurements table completed, a line or bar graph (not necessarily best-fit) drawn, and predictions and conclusions drawn from the data. There may be minor omissions, inaccuracies, or errors in the details of the data treatment and the drawing of conclusions.

Communication Skills

- A solution that may provide a data table, shows the results of any computations in a somewhat disorganized manner, and may not even include any form of concluding statement. The line or bar graph makes little attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Large gaps are left to be filled in by the reader.

Task 1: Marbles In a Measuring Cylinder

If solid objects are placed in a jar of water, the water level will rise. The more objects that are placed in the jar, the more the water level will rise. This investigation is designed to relate the rise in water level to the number of identical objects placed in the jar.

Student instructions and information

Part 1: **a.** Fill your measuring cylinder half full of water.

b. Measure the volume of the water to the nearest millilitre. Record the result in a chart.

c. Put one marble in the measuring cylinder.

d. Measure the new volume of the water and record the result in the chart.

e. Repeats steps c. and d. several times.

Part 2: Complete your chart of measurements.

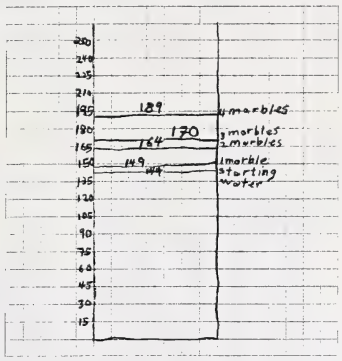
Part 3: Display your results in an appropriate graph. The graph should have measured volume on the vertical axis and the number of marbles on the horizontal axis.

Chart of measurements

Continued

Continued

Graph of water volume and number of marbles



Predictions

Average volume of one marble:

marbles/mL
1 = 5
2 = 15
3 = 6
4 = 19

Volume in the measuring cylinder if 20 marbles have been put in the water:

45 × 5 = 225 I predict the volume should increase 225 mL after 20 marbles.

Commentary

*This response receives a score of 4 for **mathematical content** because the student*

- provides a full set of measurements, using a set of marbles of different sizes, as can be seen from the data. The first marble has a volume of 5 mL, the second has a volume of 15 mL, the third a volume of 6 mL, and the fourth a volume of 19 mL
- makes a graph that is essentially a stacked bar graph, which may be an inappropriate form of graph, but nevertheless was drawn reasonably accurately
- uses ratio and proportion to predict the 225 mL water displacement for the 20 marbles, but forgets to add the initial 144 mL that was in the measuring cylinder before any marbles were added
- does not record an average volume of one marble as the marble volumes vary so much

*This response receives a score of 1 for **communication skills** because the student*

- draws a graph, but uses a stacked bar graph that does not fit in with the conventional methods of recording data such as these. Normal convention dictates the use of line graphs, or possibly bar graphs, in these circumstances
- leaves it up to the reader to estimate whether the marble volumes are different from one another, or whether the marble volumes are all equal, but have been measured carelessly

It should be noted that the student was given marbles that were different in size. Had the student obtained these measurements from marbles that were identical, the mathematical content score would have been 2, as the errors in measurement then make the whole data treatment unreliable.

This response would receive a score of	**3**	*for mathematical content*
and would receive a score of	**2**	*for communication skills*

Scoring Criteria

Mathematical Content

- The task is complete, with the measurements table completed, a line or bar graph drawn, and conclusions drawn from the data. There are significant omissions or errors in the details of the data treatment and the drawing of conclusions.

Communication Skills

- A solution that includes a data table, shows the results of any computations in some manner, and includes some form of concluding statement. The line or bar graph makes an attempt to respect the normal conventions of axis labeling, g raph drawing, and graph titling. Some gaps are left to be filled in by the reader.

Task 1: Marbles In a Measuring Cylinder

If solid objects are placed in a jar of water, the water level will rise. The more objects that are placed in the jar, the more the water level will rise. This investigation is designed to relate the rise in water level to the number of identical objects placed in the jar.

Student instructions and information

Part 1: **a.** Fill your measuring cylinder half full of water.

b. Measure the volume of the water to the nearest millilitre. Record the result in a chart.

c. Put one marble in the measuring cylinder.

d. Measure the new volume of the water and record the result in the chart.

e. Repeats steps c. and d. several times.

Part 2: Complete your chart of measurements.

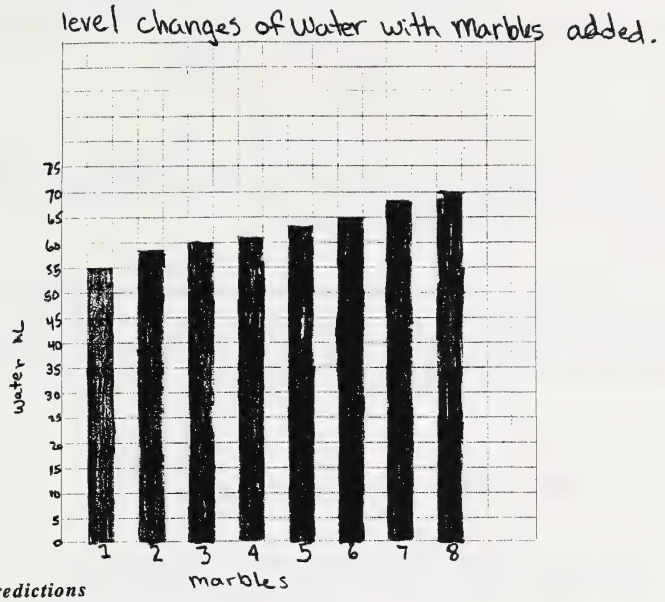
Part 3: Display your results in an appropriate graph. The graph should have measured volume on the vertical axis and the number of marbles on the horizontal axis.

Chart of measurements

Continued

Continued

Graph of water volume and number of marbles



Predictions

Average volume of one marble:

3mL

Volume in the measuring cylinder if ~~20~~ 8 marbles have been put in the water:

70mL

Commentary

*This response receives a score of 3 for **mathematical content** because the student*

- produces a bar graph that reasonably displays the data from which it was obtained. The volume for zero marbles, presumably about 52 mL, has been omitted
- makes one prediction, 3 mL for one marble, that is consistent with the data, but does not attempt to predict for 20 marbles. Instead, the student erases the 20 marbles, substitutes 8 marbles, and records the 70 mL directly from the graph
- does not display the data table from which the graph was derived

*This response receives a score of 2 for **communication skills** because the student*

- generally follows the conventions involved in the drawing of bar graphs, including the provision of units and labels on the axes, as well as a relevant title for the complete graph
- leaves it up to the reader to provide the data from which the graph was derived, and to provide the reasoning for the prediction of 3 mL for the volume of one marble

This response would receive a score of	**2**	*for mathematical content*
and would receive a score of	**1**	*for communication skills*

Scoring Criteria

Mathematical Content

- An answer that shows an ability to follow the question directions, so as to produce a measurement table, a graph, and some conclusions. However, the number of errors present in the solution render all the data treatments unreliable.

Communication Skills

- A solution that may provide a data table, shows the results of any computations in a somewhat disorganized manner, and may not even include any form of concluding statement. The line or bar graph makes little attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Large gaps are left to be filled in by the reader.

Task 1: Marbles In a Measuring Cylinder

If solid objects are placed in a jar of water, the water level will rise. The more objects that are placed in the jar, the more the water level will rise. This investigation is designed to relate the rise in water level to the number of identical objects placed in the jar.

Student instructions and information

Part 1: **a.** Fill your measuring cylinder half full of water.

b. Measure the volume of the water to the nearest millilitre. Record the result in a chart.

c. Put one marble in the measuring cylinder.

d. Measure the new volume of the water and record the result in the chart.

e. Repeats steps c. and d. several times.

Part 2: Complete your chart of measurements.

Part 3: Display your results in an appropriate graph. The graph should have measured volume on the vertical axis and the number of marbles on the horizontal axis.

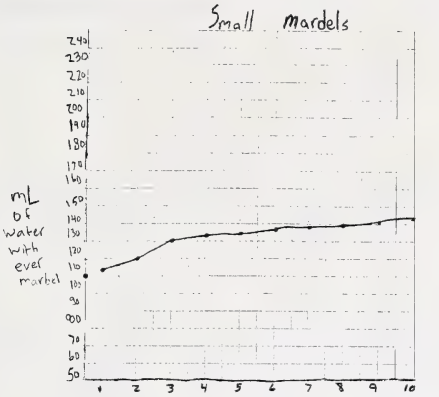
Chart of measurements

1	105
2	110
3	120
4	132
5	135
6	137
7	139
8	140
9	142
10	145

Continued

Continued

Graph of water volume and number of marbles



Predictions

Average volume of one marble:

V = 110

Volume in the measuring cylinder if 20 marbles have been put in the water:

V = 145 mL

Commentary

This response receives a score of 2 for ***mathematical content*** *because the student*

- produces a chart, giving the volumes from 1 marble to 10 marbles, and then produces a line graph that approximately reproduces these data
- makes a prediction for the volume of one marble that is unreasonable: the value of 110 mL is the initial volume of the water plus the volume of two marbles. Instead of making a prediction for 20 marbles, the volume for 10 marbles is repeated as a prediction for 20 marbles
- makes inaccurate measurements of marble volumes. It is unlikely that the first marble has a volume of 5 mL, the second one has a volume of 5 mL, and the third one has a volume of 10 mL, while all the rest have volumes of 2 mL or 3 mL each

This response receives a score of 1 for ***communication skills*** *because the student*

- leaves gaps in the chart, resulting in a chart that is difficult to follow, without labels, and without units
- makes some attempt to follow conventions in the drawing of the line graph, but puts the scale marks, such as 50 mL, in the middle of the squares, rather than with tick marks on the vertical axis. The title does not inform the reader as to what variables are being plotted on the graph

It should be noted that this response represents the upper end of the 2 criteria on the scale for mathematical content. Some more accurate readings for marble measurements would have been enough for a score of 3. The spelling errors "mardels" in the title and "marbel" on the axis labels are not a factor in assigning a score for communication skills.

This response would receive a score of	**1**	*for mathematical content*
and would receive a score of	**1**	*for communication skills*

Scoring Criteria

Mathematical Content

- A significant start made to the problem, such as (but not limited to) the setting up of a measurement table, together with one or two data points, or the drawing of a line graph, or a reasonable prediction of either the 20-marble volume or the volume of a single marble.

Communication Skills

- A solution that may provide a data table, shows the results of any computations in a somewhat disorganized manner, and may not even include any form of concluding statement. The line or bar graph makes little attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Large gaps are left to be filled in by the reader.

Task 1: Marbles In a Measuring Cylinder

If solid objects are placed in a jar of water, the water level will rise. The more objects that are placed in the jar, the more the water level will rise. This investigation is designed to relate the rise in water level to the number of identical objects placed in the jar.

Student instructions and information

Part 1: **a.** Fill your measuring cylinder half full of water.

b. Measure the volume of the water to the nearest millilitre. Record the result in a chart.

c. Put one marble in the measuring cylinder.

d. Measure the new volume of the water and record the result in the chart.

e. Repeats steps c. and d. several times.

Part 2: Complete your chart of measurements.

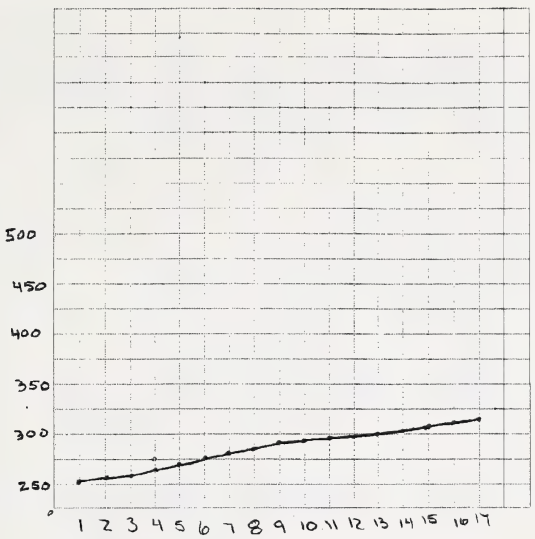
Part 3: Display your results in an appropriate graph. The graph should have measured volume on the vertical axis and the number of marbles on the horizontal axis.

Chart of measurements

Continued

Continued

Graph of water volume and number of marbles



Predictions

Average volume of one marble:

Volume in the measuring cylinder if 20 marbles have been put in the water:

Commentary

*This response receives a score of 1 for **mathematical content** because the student*

- makes the significant step of producing a line graph that represents some reasonably believable data; however, the data is not recorded separately
- does not use the graph to make any predictions from the data

*This response receives a score of 1 for **communication skills** because the student*

- produces a line graph from some unknown data set, but uses an inappropriate scale on the vertical axis. If data runs from 250 mL to 320 mL, one square can represent 5 mL and not 25 mL, and then the graph fills the space available
- omits the title from the graph, as well as the labels for both vertical and horizontal axes

Task 2: Determining the Area of a Quadrilateral Student Task and Solution

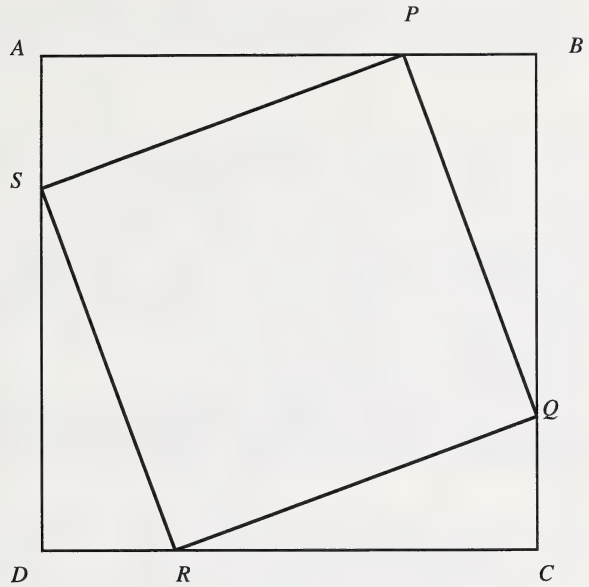
A drawing has been started on the facing page. The points $A, B, P,$ and D have been marked. The distances AB and AD are both 15 units, and the distance AP is 11 units. To complete the drawing:

1. Mark the point C so that $ABCD$ is a square.
2. Mark the point Q on BC, so that the distance BQ is 11 units.
3. Mark the point R on CD, so that the distance CR is 11 units.
4. Mark the point S on DA, so that the distance DS is 11 units.
5. Join the points $P, Q, R,$ and S.

The problem is to determine the area of the quadrilateral $PQRS$ in **two** different ways.

Finished Work

Drawing:



Calculations (remember to show your work):

Area of quadrilateral *PQRS*:

Solution

The area is the area of square *ABCD* less four times the area of triangle *QCR*.

So area is (15)(15) – 4(0.5)(11)(4) or 225 – 88 or **137 square units**

Area of quadrilateral *PQRS* (using an alternative method):

Solution

The area is the square of the distance *PQ*.

Use Pythagoras to find PQ^2. Here $PQ^2 = PB^2 + BQ^2$

So $PQ^2 = 4^2 + 11^2$, or **137 square units**

By counting squares directly, the area would be somewhere between **130 and 140 square** units, depending on whether each partial square is interpolated, or is automatically counted as a half-square.

Task-Specific Scoring Criteria

Mathematical Content Scale

Scale score	Criteria
5	A complete and correct solution, consisting of an accurate drawing, a clear indication of the shape of *PQRS*, and the area calculated in two distinct ways.
4	**Either** a correct solution consisting of an accurate drawing, including some description of the shape of *PQRS*, but with the area calculated in only one way; **or** a complete solution, consisting of an accurate drawing, a description of the shape of *PQRS*, and the area calculated in two distinct ways. There are minor errors in measurement or calculation present.
3	**Either** a solution, containing minor errors, consisting of an accurate drawing, an implied description of the shape of *PQRS*, but with the area calculated in only one way; **or** a complete solution, consisting of a drawing, a description of the shape of *PQRS*, and the area calculated in two distinct ways. There are significant errors in measurement and calculation present.
2	**Either** a solution that consists of an accurate drawing, with no significant progress in the area calculations; **or** a reasonable attempt to calculate the area *PQRS* in two distinct ways, but starting from an inaccurate or incomplete drawing.
1	A significant start on the problem. Examples of significant starts include (but are not limited to) drawing a square *ABCD* of the correct shape and dimensions, or using formulas such as the area of a rectangle, area of a triangle, or Pythagoras' Theorem, or assigning a consistent name to the shape of the quadrilateral *PQRS*.
0	**Either** off-topic; **or** a blank paper.

Continued

Mathematics 14 – Continued

Communication Skills Scale

This scale is used **independently** of the mathematical content scale. A well-drawn, fully labeled drawing can receive a scale score of 2, even if the supporting calculations are incorrect, or insufficiently well explained.

Scale Score	Criteria
3	A clear solution that includes a completed drawing that generally respects the normal conventions of labeling of points, and of line drawing. The solution shows the results of any computations in a systematic, ordered manner, and includes full concluding statements. Few, if any, gaps are left in drawings or calculations.
2	A solution that includes a completed drawing that makes an attempt to respect the normal conventions of labeling of points, and of line drawing. The solution shows the results of any computations in some manner, and may include some form of concluding statement. Some gaps, in both drawing and calculation, are left to be filled in by the reader.
1	A solution that may provide a completed drawing that makes little attempt to respect the normal conventions of labeling of points, and of line drawing. The solution shows the results of any computations in a somewhat disorganized manner, and may not even include any form of concluding statement. Large gaps are left to be filled in by the reader.
0	**Either** insufficient evidence of communication skills; **or** a blank paper.

Task 2 : Student Responses

This response would receive a score of	**5**	*for mathematical content*
and would receive a score of	**2**	*for communication skills*

Scoring Criteria

Mathematical Content

- A complete and correct solution, consisting of an accurate drawing, a clear indication of the shape of *PQRS*, and the area calculated in two distinct ways.

Communication Skills

- A solution that includes a completed drawing that makes an attempt to respect the normal conventions of labeling of points, and of line drawing. The solution shows the results of any computations in some manner, and may include some form of concluding statement. Some gaps, in both drawing and calculation, are left to be filled in by the reader.

Task 2: Determining the Area of a Quadrilateral

A drawing has been started on the facing page. The points *A*, *B*, *P* and *D* have been marked. The distances *AB* and *AD* are both 15 squares, and the distance *AP* is 11 squares. To complete the drawing:

1. Mark the point *C* so that *ABCD* is a square.
2. Mark the point *Q* on *BC*, so that the distance *BQ* is 11 squares.
2. Mark the point *R* on *CD*, so that the distance *CR* is 11 squares.
4. Mark the point *S* on *DA*, so that the distance *DS* is 11 squares.
5. Join the points *P*, *Q*, *R*, and *S*.

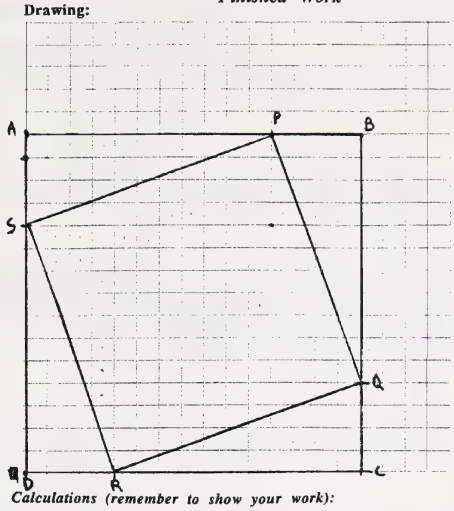
The problem is to determine the area of the quadrilateral *PQRS* in **two** different ways.

Continued

Continued

Finished Work

Drawing:



Calculations (remember to show your work):

Area of quadrilateral *PQRS*:

132 squares

Area of quadrilateral *PQRS* (using an alternative method):

A = b x h
A = 7.35cm x 7.35
A = 54.0225

Commentary

This response receives a score of 5 for ***mathematical content*** *because the student*

- completes the drawing as instructed, with the final drawing being both complete and accurate
- calculates the area first by counting squares and expressing the answer as 132 squares. **The actual dots used may not be completely visible on the photoscans used in this document**
- calculates the area a second way by using the rectangle area formula, measuring the lengths *PQ* and *QR* using a centimetre ruler, and then computing the area in square centimetres

This response receives a score of 2 for ***communication skills*** *because the student*

- follows the conventions of scale drawing in the use of rulers, the labeling of points, and the description of lines and quadrilaterals
- shows the counting of squares on the diagram to justify the first answer. The use of the unit *squares* reinforces this method of calculation
- shows the formula used for the area of the rectangle, and the units of centimetres used in the second line of the second answer
- omits the unit cm^2 in the last line of the second answer, and does not explain the switch from square units to square centimetres in the alternative solution of the area

This response would receive a score of	**4**	*for mathematical content*
and would receive a score of	**3**	*for communication skills*

Scoring Criteria

Mathematical Content

- A correct solution consisting of an accurate drawing, including some description of the shape of *PQRS*, but with the area calculated in only one way.

Communication Skills

- A clear solution that includes a completed drawing that generally respects the normal conventions of labeling of points, and of line drawing. The solution shows the results of any computations in a systematic, ordered manner, and includes full concluding statements. Few, if any, gaps are left in drawings or calculations.

Task 2: Determining the Area of a Quadrilateral

A drawing has been started on the facing page. The points *A*, *B*, *P* and *D* have been marked. The distances *AB* and *AD* are both 15 squares, and the distance *AP* is 11 squares. To complete the drawing:

1. Mark the point *C* so that *ABCD* is a square.
2. Mark the point *Q* on *BC*, so that the distance *BQ* is 11 squares.
2. Mark the point *R* on *CD*, so that the distance *CR* is 11 squares.
4. Mark the point *S* on *DA*, so that the distance *DS* is 11 squares.
5. Join the points *P*, *Q*, *R*, and *S*.

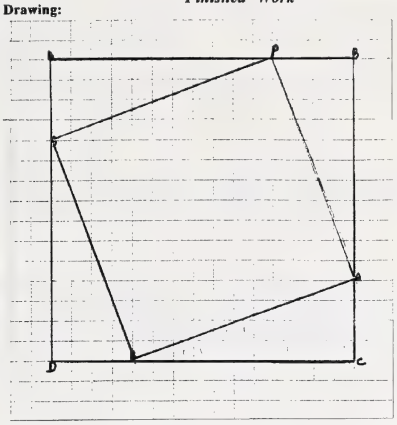
The problem is to determine the area of the quadrilateral *PQRS* in **two** different ways.

Continued

Continued

Finished Work

Drawing:



Calculations (remember to show your work):

Area of quadrilateral *PQRS*:

137

Area of quadrilateral *PQRS* (using an alternative method):

15 x 15 = 225

a corner not in PQRS is 22

22 x 4 = 88

88 - 225 = 137

Commentary

*This response receives a score of 4 for **mathematical content** because the student*

- completes the diagram accurately, following the directions, and labeling a clear diagram that shows both the square *ABCD* and the quadrilateral *PQRS*
- calculates the area of quadrilateral *PQRS* as that of the square *ABCD* – four times the area of triangle *QCR*, or 225 – (4 × 22) = 137
- presumes that the first answer for the area of *PQRS* is 137, without giving any indication of using a second way of calculating this area

*This response receives a score of 3 for **communication skills** because the student*

- follows the conventions of scale drawing in the use of rulers, the labeling of points, and the description of lines and quadrilaterals
- shows the counting of squares and the formulas used for the area of the rectangle and the triangles on the diagram to justify the second answer. The limited use of words to explain the process enhances the clarity somewhat
- shows no method of obtaining the first answer; however, since no credit was given for this part of the response for mathematical content, the extreme brevity of the response can be disregarded in assessing communication skills

This response would receive a score of	**3**	*for mathematical content*
and would receive a score of	**2**	*for communication skills*

Scoring Criteria

Mathematical Content

- A solution containing minor errors consisting of an accurate drawing, an implied description of the shape of *PQRS*, but with the area calculated in only one way.

Communication Skills

- A solution that includes a completed drawing that makes an attempt to respect the normal conventions of labeling of points, and of line drawing. The solution shows the results of any computations in some manner, and may include some form of concluding statement. Some gaps, in both drawing and calculation, are left to be filled in by the reader.

Task 2: Determining the Area of a Quadrilateral

A drawing has been started on the facing page. The points *A*, *B*, *P* and *D* have been marked. The distances *AB* and *AD* are both 15 squares, and the distance *A*P is 11 squares. To complete the drawing:

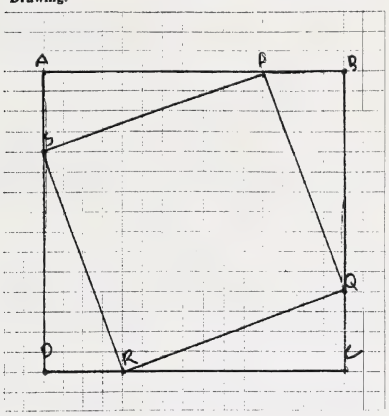
1. Mark the point *C* so that *ABCD* is a square.
2. Mark the point *Q* on *BC*, so that the distance *BQ* is 11 squares.
2. Mark the point *R* on *CD*, so that the distance *CR* is 11 squares.
4. Mark the point *S* on *DA*, so that the distance *DS* is 11 squares.
5. Join the points *P*, *Q*, *R*, and *S*.

The problem is to determine the area of the quadrilateral *PQRS* in **two** different ways.

Continued

Continued

Drawing: *Finished Work*



Calculations (remember to show your work):

Area of quadrilateral *PQRS*:

9.5
7.5 9.5 7.5 139.24 units
15 11.8

Area of quadrilateral *PQRS* (using an alternative method):

Commentary

This response receives a score of 3 for ***mathematical content*** *because the student*

- completes the diagram accurately, following the directions, and labeling a clear diagram that shows both the square *ABCD* and the quadrilateral *PQRS*
- calculates the area of quadrilateral *PQRS* as some ratio of length, related to the area of the initial square *ABCD*. The final answer of 139.24 units is sufficiently close to the accurate answer of 137 units to be judged a valid solution
- only uses one method of calculation of the area of the quadrilateral *PQRS*

This response receives a score of 2 for ***communication skills*** *because the student*

- produces a drawing that is complete, accurate, uses appropriate drawing instruments, and whose labeling indicates a respect for the needs of the reader in the production and the drawing. There are no gaps here that the reader need fill in
- produces a calculation that uses ratio and proportion, showing two fractions $\frac{9.5}{15}$ and $\frac{7.5}{11.8}$; there is no explanation for these fractions, or for the number 139.24

Here, the communication skills score is more of an average, as the drawing clearly satisfies the criteria for a score of 3, while the calculation, with its large gaps in communication, is closer to satisfying the criteria for a score of 1. All the gaps in communication are in the calculations segment of the response.

This response would receive a score of	**2**	*for mathematical content*
and would receive a score of	**1**	*for communication skills*

Scoring Criteria

Mathematical Content

- A solution that consists of an accurate drawing, with no significant progress in the area calculations.

Communication Skills

- A solution that may provide a completed drawing, shows the results of any computations in a somewhat disorganized manner, and may not even include any form of concluding statement. The drawing makes little attempt to respect the normal conventions of labeling of points, and of line drawing. Large gaps are left to be filled in by the reader.

Task 2: Determining the Area of a Quadrilateral

A drawing has been started on the facing page. The points *A*, *B*, *P* and *D* have been marked. The distances *AB* and *AD* are both 15 squares, and the distance *AP* is 11 squares. To complete the drawing:

1. Mark the point *C* so that *ABCD* is a square.
2. Mark the point *Q* on *BC*, so that the distance *BQ* is 11 squares.
2. Mark the point *R* on *CD*, so that the distance *CR* is 11 squares.
4. Mark the point *S* on *DA*, so that the distance *DS* is 11 squares.
5. Join the points *P*, *Q*, *R*, and *S*.

The problem is to determine the area of the quadrilateral *PQRS* in two different ways.

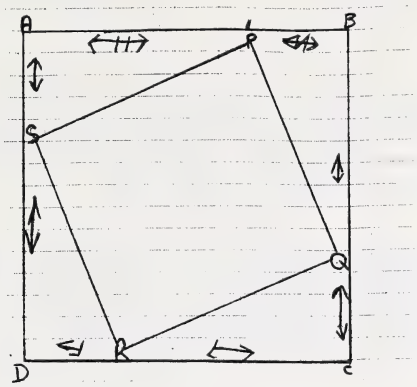
27 cm

Continued

Continued

Finished Work

Drawing:



Calculations (remember to show your work):

Area of quadrilateral *PQRS*:

↕6.5 cm, 7cm, 6.5↕ 7cm ↔ = 27

Area of quadrilateral *PQRS* (using an alternative method):

Commentary

This response receives a score of 2 for ***mathematical content*** *because the student*

- completes the diagram fairly accurately, following the directions, and labels a diagram that shows both the square *ABCD* and the quadrilateral *PQRS*. The labels for *P*, *Q*, *R* and *S* are placed in the correct region, rather than at an exact point
- measures four lengths, two as 7 cm and two as 6.5 cm, and then calculates a value of 27 as the area, when the 27 is more closely related to a perimeter

This response receives a score of 1 for ***communication skills*** *because the student*

- produces a drawing that is complete and fairly accurate, uses a very thick pencil, and whose labeling indicates the qualities of a sketch, rather than a production drawing
- produces a calculation that uses the units of centimetres, rather than the natural units of squares, without attempting to explain the reasons for the switch in units. There are significant gaps, such as the ambiguous labeling of ↕ for both the 6.5 cm lengths and ↔ for both the 7 cm lengths; none of the line segments *PQ*, *QR*, *RS*, and *SP* are either vertical or horizontal, and the symbols ↕ and ↔ are attached to line segments such as *DS* and *RC*

This response would receive a score of	**1**	*for mathematical content*
and would receive a score of	**2**	*for communication skills*

Scoring Criteria

Mathematical Content

- A significant start on the problem. Examples of significant starts include (but are are not limited to) drawing a square *ABCD* of the correct shape and dimensions, or using formulas such as the area of a rectangle, area of a triangle, or Pythagoras' Theorem, or assigning a consistent name to the shape of the quadrilateral *PQRS*.

Communication Skills

- A solution that includes a completed drawing that makes an attempt to respect the normal conventions of labeling of points, and of line drawing. The solution shows the results of any computations in some manner, and may include some form of concluding statement. Some gaps, in both drawing and calculation, are left to be filled in by the reader.

Task 2: Determining the Area of a Quadrilateral

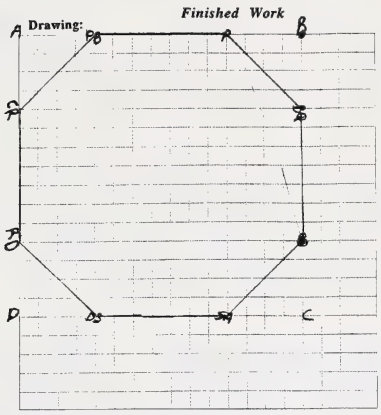
A drawing has been started on the facing page. The points *A*, *B*, *P* and *D* have been marked. The distances *AB* and *AD* are both 15 squares, and the distance *AP* is 11 squares. To complete the drawing:

1. Mark the point *C* so that *ABCD* is a square.
2. Mark the point *Q* on *BC*, so that the distance *BQ* is 11 squares.
2. Mark the point *R* on *CD*, so that the distance *CR* is 11 squares.
4. Mark the point *S* on *DA*, so that the distance *DS* is 11 squares.
5. Join the points *P*, *Q*, *R*, and *S*.

The problem is to determine the area of the quadrilateral *PQRS* in **two** different ways.

Continued

Continued



Calculations (remember to show your work):

Area of quadrilateral *PQRS*:

$A = l \times w$

$A = (9.5\text{cm})(9.5\text{cm})$

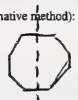
$A = 90.25\text{ cm}^2$

Area of quadrilateral *PQRS* (using an alternative method):

$A = \frac{2h(a+h)}{2\times 2}$

$A = \frac{2\times 9.5\,(9.5+9.5)}{4}$

$A = \frac{19\,(19)}{4}$ → $A = 90.25$



cut in half

Commentary

This response receives a score of 1 for ***mathematical content*** *because the student*

- starts with an octagon for the quadrilateral *PQRS*, rather than a square. The 11 and the 4 are recorded on the diagram, but the points are taken both clockwise and anticlockwise around the original square. As a result, there are two intermediate points, labeled *PB* and *P* on side *AB*; two, labeled *BQ* and some unreadable letters, on side *BC*, and so on around the original square
- calculates an area of 90.25 cm^2, which refers to the area of the square *ABCD* in cm^2, rather than the area of the octagon in square units
- calculates the area of an octagon using an incorrect formula, in which the variables are undefined

This response receives a score of 2 for ***communication skills*** *because the student*

- produces a drawing that is generally consistent with the conventions of line drawing, but whose labeling of points with two letters such as *BQ* or with unreadable letters, detracts slightly from its clarity
- expresses calculations in terms of square centimetres, and calculates in terms of unfamiliar measurement formulas that are expressed in terms of appropriate variables

This response would receive a score of	**1**	*for mathematical content*
and would receive a score of	**1**	*for communication skills*

Scoring Criteria

Mathematical Content

- A significant start on the problem. Examples of significant starts include (but are not limited to) drawing a square *ABCD* of the correct shape and dimensions, or using formulas such as the area of a rectangle, area of a triangle, or Pythagoras' Theorem, or assigning a consistent name to the shape of the quadrilateral *PQRS*.

Communication Skills

- A solution that may provide a completed drawing, shows the results of any computations in a somewhat disorganized manner, and may not even include any form of concluding statement. The drawing makes little attempt to respect the normal conventions of labeling of points, and of line drawing. Large gaps are left to be filled in by the reader.

Task 2: Determining the Area of a Quadrilateral

A drawing has been started on the facing page. The points *A*, *B*, *P* and *D* have been marked. The distances *AB* and *AD* are both 15 squares, and the distance *AP* is 11 squares. To complete the drawing:

1. Mark the point *C* so that *ABCD* is a square.
2. Mark the point *Q* on *BC*, so that the distance *BQ* is 11 squares.
2. Mark the point *R* on *CD*, so that the distance *CR* is 11 squares.
4. Mark the point *S* on *DA*, so that the distance *DS* is 11 squares.
5. Join the points *P*, *Q*, *R*, and *S*.

The problem is to determine the area of the quadrilateral *PQRS* in **two** different ways.

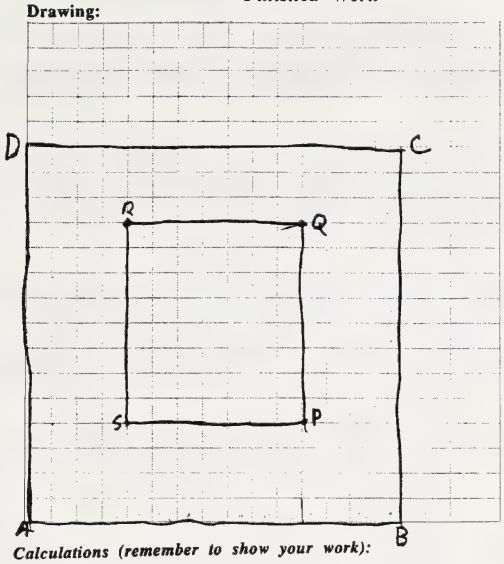
Square

Continued

Continued

Finished Work

Drawing:



Calculations (remember to show your work):

Area of quadrilateral *PQRS*:

56 Squares = 7 × 6

Area of quadrilateral *PQRS* (using an alternative method):

Commentary

This response receives a score of 1 for ***mathematical content*** *because the student*

- makes the significant remark on the first page that quadrilateral *PQRS* is a square. The drawing on the second page does not reflect this as line *QR* is drawn 3 squares below line *CD*, in contrast to lines *SP*, *PQ*, and *RS*, which are all drawn 4 squares in from the sides *AB*, *BC*, and *DA* respectively
- does not realize that *P* lies on *AB*, and similarly *Q* lies on *BC*, and so on
- makes a consistent count of the area *PQRS* as 56 squares from the diagram, but then expresses this as 7×6

This response receives a score of 1 for ***communication skills*** *because the student*

- makes some effort to respect the rules of drawing and labeling of diagrams, although the quality of the drawing is that of a rough freehand, rather than the expected drawing using a ruler and pencil
- leaves considerable gaps in the calculation segment

The discussion of Task 3 starts on page 69

Task 3: Does One 12-Sided Die Equal Two 6-Sided Dice? Student Task and Solution

Procedure

Throw the 12-sided die 50 times and record the results in the third column of the first tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Throw the pair of 6-sided dice 50 times and record the results in the third column of the second tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Draw a frequency graph or histogram from **each** one of the tally charts.

Tally Charts

Solution

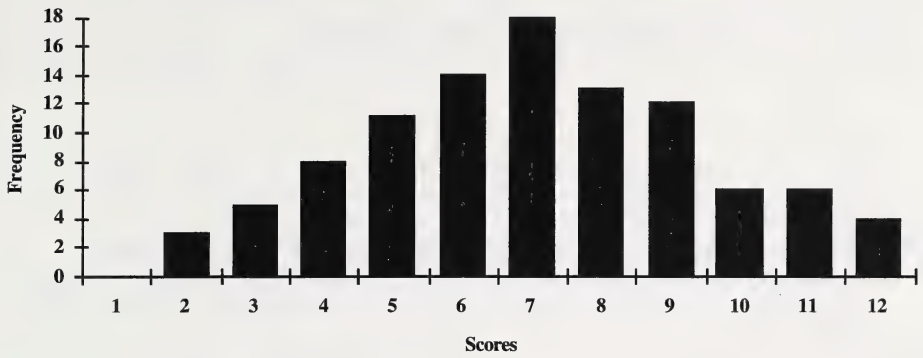
Chart 1: 12-sided die			
Score (12-sided die)	Frequency (first 50)	Tallies for second 50	Total frequency
1	3	11111 11	10
2	5	1111	9
3	4	1111	8
4	4	11111 1	10
5	6	1	7
6	3	11111	8
7	5	1	6
8	3	11111 1	9
9	2	11111	7
10	6	11	8
11	5	1111	9
12	4	11111	9

Chart 2: Pair of 6-sided dice			
Score (6-sided dice)	**Frequency (1st 50)**	**Tallies for second 50**	**Total frequency**
1	0		0
2	2	1	3
3	2	111	5
4	3	11111	8
5	6	11111	11
6	7	11111 11	14
7	9	11111 1111	18
8	7	11111 1	13
9	5	11111 11	12
10	5	1	6
11	3	111	6
12	1	111	4

Frequency Graph or Histograms

Frequency Graph for 100 Rolls of One 12-Sided Die

Frequency Graph for 100 Rolls of Two 6-Sided Dice



Conclusions

Use one or two sentences to describe how the graphs look the same, or how the graphs look different.

Solution

The pair of 6-sided dice give a pattern that favors scores in the middle of the range, such as 6, 7, and 8. Also the score of 1 is impossible.

The 12-sided die gives a more random pattern, and the score of 1 is possible

Task-Specific Scoring Criteria for Task 3

Mathematical Content Scale

Scale score	Criteria
5	A complete and correct solution, with both tally charts completed, frequency graphs drawn, and conclusions made that follow from the data obtained.
4	The task is complete, with both tally charts completed, frequency graphs drawn,but with no conclusions drawn from the data. There are minor omissions or errors in the details of the data treatment, but the omissions or errors can be easily rectified.
3	**Either** the task is complete, with both tally charts completed, frequency graphs drawn, but with no conclusions drawn from the data. There are significant omissions or errors in the details of the data treatment and the drawing of conclusions; **or** a completed, correct set of tally charts, together with a start made on the frequency graphs, and little attempt to draw any systematic conclusions.
2	**Either** a completed, correct set of tally charts; **or** an answer that shows an ability to follow the question directions, producing tally charts, frequency graphs, and conclusions. However, the number of errors present in the solution render all the data treatments unreliable.
1	A significant start made to the problem, such as (but not limited to) the completion of one observation chart, or the identification of crucial facts, like the rolling of a 1 only being possible for the 12-sided die, or the conversion of any form of observation chart into a frequency graph.
0	**Either** off-topic; **or** a blank paper.

Continued

Mathematics 14 – Continued

Communication Skills Scale

This scale is used **independently** of the mathematical content scale. A well-laid-out, systematic strategy with conclusions clearly stated can receive a scale score of 3, even if the comparison itself is based on data that either contain gross errors in the calculations of the supporting numbers, or reflect serious errors in data recording

Scale score	Criteria
3	A clear solution that includes both tally charts, and includes full concluding statements. The bar, line, or pie graphs generally respect the normal conventions of axis labeling, graph drawing, and graph titling. Few, if any, gaps are left to be filled in by the reader.
2	A solution that includes both tally charts, and includes some form of concluding statement. The bar, line, or pie graphs make an attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Some gaps are left to be filled in by the reader.
1	A solution that may provide one or both tally charts, and may not even include any form of concluding statement. The bar, line, or pie graphs make little attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Large gaps are left to be filled in by the reader.
0	**Either** insufficient evidence of communication skills; **or** a blank paper.

In the student responses that follow, any unused graphing grids have been cut to save space. There is no student writing on any of the grids that have been cut.

Task 3 : Student Responses

This response would receive a score of	**5**	*for mathematical content*
and would receive a score of	**3**	*for communication skills*

Scoring Criteria

Mathematical Content

- A complete and correct solution, with both tally charts completed, frequency graphs drawn, and conclusions made that follow from the data obtained.

Communication Skills

- A clear solution that includes both tally charts, and includes full concluding statements. The bar, line, or pie graphs generally respect the normal conventions of axis labeling, graph drawing, and graph titling. Few, if any, gaps are left to be filled in by the reader.

Task 3: Does One 12-Sided Die Equal Two 6-Sided Dice?

Procedure

Throw the 12-sided die 50 times, and record the results in the third column of the first tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Throw the pair of 6-sided dice 50 times, and record the results in the third column of the second tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Draw a frequency graph or histogram from **each** one of the tally charts.

From your tally charts and your graphs, answer the following question:

- Are the shapes of the graphs the same or different for the two experiments?

Tally Charts

Chart 1: 12-sided die:

Score (12-sided die)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	3	𝍸	8
2	5	𝍸	9
3	4	𝍸	10
4	4	𝍸	10
5	6	𝍸	11
6	3	𝍸	6
7	5	𝍸	9
8	3	𝍸	8
9	2	𝍸	10
10	6	𝍸	10
11	5	𝍸	10
12	4	𝍸	6

Continued

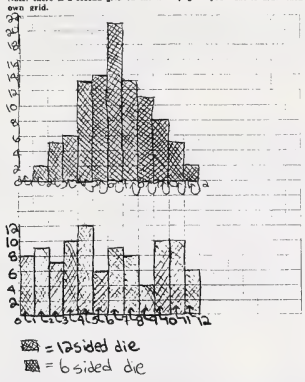
Continued

Chart 2: Pair of 6-sided dice:

Score (6-sided dice)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	0		0
2	2		2
3	2		5
4	3		6
5	6		13
6	7		14
7	9		21
8	7		13
9	5		11
10	5		9
11	3		5
12	1		2

Frequency Graphs or Histograms

Note: there is a second grid on the next page if you want to draw each graph on its own grid.



= 12 sided die

= 6 sided die

Conclusions

Compare and contrast the shapes of the two graphs. Use one or two sentences to describe how the graphs look the same, or how the graphs look different.

and they are different, one goes ^ the other doesn't.

the 12 sided die chart is higher. (as an average score)

Commentary

*This response receives a score of 5 for **mathematical content** because the student*

- completes both tally charts, and produces accurate frequency graphs that relate to the data produced
- draws one major point of difference in the shapes of the two frequency graphs. The conclusion may be stated colloquially with a very rough diagram, but is nevertheless completely correct
- draws one minor point of difference in the average score, though this difference is not backed up with numerical calculations

*This response receives a score of 3 for **communication skills** because the student*

- produces tally charts and frequency graphs that are easy to read and whose construction is consistent with graphing conventions
- puts the two frequency graphs next to one another, so that the reader can easily see the difference, and uses different shading to distinguish between the two graphs
- uses a diagram to provide an accurate summary of the essential difference in shape between the two graphs

This response would receive a score of	**4**	*for mathematical content*
and would receive a score of	**2**	*for communication skills*

Scoring Criteria

Mathematical Content

- The task is complete, with both tally charts completed, frequency graphs drawn, but with no conclusions drawn from the data. There are minor omissions or errors in the details of the data treatment, but the omissions or errors can be easily rectified.

Communication Skills

- A solution that includes both tally charts, and includes some form of concluding statement. The bar, line, or pie graphs make an attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Some gaps are left to be filled in by the reader.

Task 3: Does One 12-Sided Die Equal Two 6-Sided Dice?

Procedure

Throw the 12-sided die 50 times, and record the results in the third column of the first tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Throw the pair of 6-sided dice 50 times, and record the results in the third column of the second tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Draw a frequency graph or histogram from **each** one of the tally charts.

From your tally charts and your graphs, answer the following question:

- Are the shapes of the graphs the same or different for the two experiments?

Tally Charts

Chart 1: 12-sided die:

Score (12-sided die)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	3	𝍸	8
2	5	𝍸	10
3	4	𝍸	7
4	4	𝍸	9
5	6	𝍸	10
6	3	𝍸 ||	10
7	5	||	7
8	3	||||	7
9	2	𝍸 |||	10
10	6	|||	9
11	5	|	6
12	4	|||	7

Continued

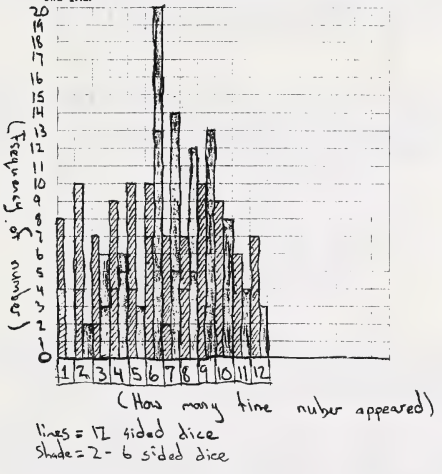
Continued

Chart 2: Pair of 6-sided dice:

Score (6-sided dice)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	0	0	0
2	2	0	2
3	2	𝍸	6
4	3	𝍸 I	9
5	6	III	9
6	7	𝍸 𝍸 III	20
7	9	𝍸	14
8	7	𝍸	12
9	5	𝍸 III	13
10	5	III	8
11	3	I	4
12	1	II	3

Frequency Graphs or Histograms

Note: there is a second grid on the next page if you want to draw each graph on its own grid.



Conclusions

Compare and contrast the shapes of the two graphs. Use one or two sentences to describe how the graphs look the same, or how the graphs look different.

Commentary

This response receives a score of 4 for ***mathematical content*** *because the student*

- completes both tally charts, and produces an accurate frequency graphs of the combined data that relates to the data contained in the tally charts
- leaves it up to the reader to draw the conclusions from the frequency graphs, rather than providing a summary for the reader

This response receives a score of 2 for ***communication skills*** *because the student*

- completes the tally charts in a clear easy-to-read format
- completes the frequency graphs, with axes labeled appropriately, and frequency on the vertical axis
- puts both frequency graphs on the same set of axes, making the columns difficult to read. Splitting the bar representing a score of 4 into two thin pieces, one for the 12-sided die, and the other for the pair of 6-sided dice, makes the graph very difficult to read and to interpret

This response would receive a score of	**3**	*for mathematical content*
and would receive a score of	**2**	*for communication skills*

Scoring Criteria

Mathematical Content

- The task is complete, with both tally charts completed, frequency graphs drawn, and but with no conclusions drawn from the data. There are significant omissions or errors in the details of the data treatment and the drawing of conclusions.

Communication Skills

- A solution that includes both tally charts, and includes some form of concluding statement. The bar, line, or pie graphs make an attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Some gaps are left to be filled in by the reader.

Task 3: Does One 12-Sided Die Equal Two 6-Sided Dice?

Procedure

Throw the 12-sided die 50 times, and record the results in the third column of the first tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Throw the pair of 6-sided dice 50 times, and record the results in the third column of the second tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Draw a frequency graph or histogram from **each** one of the tally charts.

From your tally charts and your graphs, answer the following question:

- Are the shapes of the graphs the same or different for the two experiments?

Tally Charts

Chart 1: 12-sided die:

Score (12-sided die)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	3	|||	3
2	5	|||| |||| |	6
3	4	|||| ||||	6
4	4	|||	3
5	6	||||	4
6	3	||| |	4
7	5	|||| ||||	5
8	3	|| |||	4
9	2	|||| ||||	6
10	6	||||	3
11	5	||||	3
12	4	||||	3

Continued

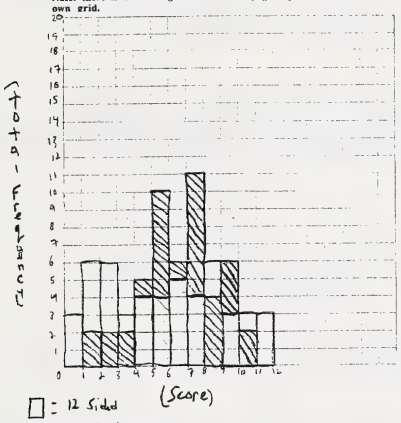
Continued

Chart 2: Pair of 6-sided dice:

Score (6-sided dice)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	0		0
2	2		2
3	2		2
4	3		2
5	6		5
6	7		10
7	9		6
8	7		11
9	5		4
10	5		6
11	3		2
12	1		0

Frequency Graphs or Histograms

Note: there is a second grid on the next page if you want to draw each graph on its own grid.



Conclusions

Compare and contrast the shapes of the two graphs. Use one or two sentences to describe how the graphs look the same, or how the graphs look different.

Commentary

*This response receives a score of 3 for **mathematical content** because the student*

- produces a tally chart for the 12-sided die only for the second group of 50 tosses, and just transfers the check marks in the third column to numbers in the fourth column of each tally chart. The tallies in the third column should have been added to the frequencies in the second column to produce larger totals.
- produces accurate frequency graphs that relate to the data produced
- leaves it up to the reader to draw the conclusions from the frequency graphs, rather than providing a summary for the reader

*This response receives a score of 2 for **communication skills** because the student*

- completes the tally charts in a clear easy-to-read format
- completes the frequency graphs, with axes labeled appropriately, and frequency on the vertical axis
- stacks both frequency graphs on the same set of axes, making the columns difficult to read. Stacking the bar representing a score of 4 into two pieces, one for the 12-sided die, and the other for the pair of 6-sided dice, makes the graph very difficult to read. It is hard to see which column refers to which die or dice when reading the graphs

This response would receive a score of	**3**	*for mathematical content*
and would receive a score of	**1**	*for communication skills*

Scoring Criteria

Mathematical Content

- The task is complete, with both tally charts completed, frequency graphs drawn, and but with no conclusions drawn from the data. There are significant omissions or errors in the details of the data treatment and the drawing of conclusions.

Communication Skills

- A solution that may provide one or both tally charts, and may not even include any form of concluding statement. The bar, line, or pie graphs make little attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Large gaps are left to be filled in by the reader.

Task 3: Does One 12-Sided Die Equal Two 6-Sided Dice?

It is well known that 6 + 6 = 12, but does one 12-sided die equal two 6-sided dice when they are thrown a number of times?

Student instructions and information

Throw the 12-sided die 50 times, and record the results in the tally chart below. The result to be recorded is the number appearing on the top face. Add these results to the results of the 50 throws that have already been entered in the observation chart.

Throw the pair of 6-sided dice 50 times, and record the results in a second tally chart. The result to be recorded is the total of the two numbers appearing on the top faces, so a throw which results in a 4 appearing on one die and a 5 on the other is recorded as 4 + 5, or 9. The number 9 is recorded in the chart. Add these results to the results of the 50 throws that have already been entered in the observation chart.

Draw a frequency graph from each of the tally charts.

From your tally charts and your frequency graphs, answer the following questions:

- Are the shapes of the frequency graphs the same or different for the two experiments?
- Which experiment (12-sided die or pair of 6-sided dice) gives the higher average score?

Tally Charts

Score (12-sided die)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	3	| |	5
2	5	𝍸 | |	12
3	4	𝍸 |	10
4	4	𝍸	9
5	6	|||	9
6	3	||	5
7	5	||	7
8	3	||	5
9	2	𝍸 |||	10
10	6	𝍸 |	13
11	5	|||	8
12	4	|||	8

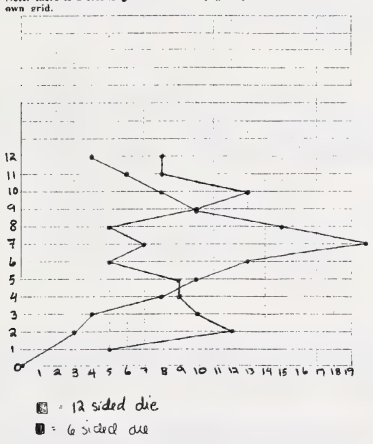
Continued

Continued

Chart 2: Pair of 6-sided dice:

Score (6-sided dice)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	0		0
2	2	|	3
3	2	||	4
4	3	||||	8
5	6	||||	10
6	7	|||||	13
7	9	|||||||||	20
8	7	||||||||	15
9	5	|||||	10
10	5	|||	8
11	3	|||	6
12	1	|||	4

Frequency Graphs or Histograms
Note: there is a second grid on the next page if you want to draw each graph on its own grid.



Conclusions

Compare and contrast the shapes of the two graphs. Use one or two sentences to describe how the graphs look the same, or how the graphs look different.

Commentary

*This response receives a score of 3 for **mathematical content** because the student*

- completes both tally charts, and produces frequency graphs that relate to the data produced
- includes extra points and lines, such as the line joining (0, 0) with the point (3, 2) on the 6-sided dice frequency line graph. The 6-sided dice graph should start from a score of 2, as scores of 0 and 1 are both impossible
- leaves a difficult task for the reader to draw the conclusions from the frequency graphs, rather than providing a summary for the reader. The difficulty is magnified by the unconventional placement of the frequencies on the horizontal, rather than the vertical, axis

*This response receives a score of 1 for **communication skills** because the student*

- provides a clear set of tally charts to summarize the data
- uses an unconventional method of putting frequencies on the horizontal axis, and makes it even harder to read by omitting both sets of axis labels
- tries to use different shapes for legends to distinguish between the two graphs. However, there is insufficient contrast between the two legends. When combined with the lack of axis labels, this makes the graph difficult to read and to interpret

This response would receive a score of	**2**	*for mathematical content*
and would receive a score of	**2**	*for communication skills*

Scoring Criteria

Mathematical Content

- An answer that shows an ability to follow the question directions, producing tally charts, frequency graphs, and conclusions. However, the number of errors present in the solution render all the data treatments unreliable.

Communication Skills

- A solution that includes both tally charts, and includes some form of concluding statement. The bar, line, or pie graphs make an attempt to respect the normal conventions of axis labeling, graph drawing, and graph titling. Some gaps are left to be filled in by the reader.

Task 3: Does One 12-Sided Die Equal Two 6-Sided Dice?

Procedure

Throw the 12-sided die 50 times, and record the results in the third column of the first tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Throw the pair of 6-sided dice 50 times, and record the results in the third column of the second tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Draw a frequency graph or histogram from **each** one of the tally charts.

From your tally charts and your graphs, answer the following question:

- Are the shapes of the graphs the same or different for the two experiments?

Tally Charts

Chart 1: 12-sided die:

Score (12-sided die)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	3	|| 2	5
2	5	||| 3	8
3	4	𝍸 𝍸 10	14
4	4	| 1	5
5	6	||| 3	9
6	3	||| 4	7
7	5	|| 2	7
8	3	𝍸 ||| 8	11
9	2	𝍸 | 6	8
10	6	𝍸 5	11
11	5	||| 4	9
12	4	|| 2	6

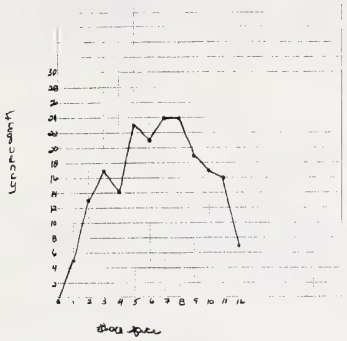
Continued

Continued

Chart 2: Pair of 6-sided dice:

Score (6-sided dice)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	0	0	0
2	2	||| 3	5
3	2	| 1	3
4	3	|||| | 6	9
5	6	|||| ||| 8	14
6	7	|||| || 7	14
7	9	|||| ||| 8	17
8	7	|||| | 6	13
9	5	|||| 6	11
10	5	| 1	6
11	3	||| 4	7
12	1		1

Frequency Graphs or Histograms
Note: there is a second grid on the next page if you want to draw each graph on its own grid.



Conclusions

Compare and contrast the shapes of the two graphs. Use one or two sentences to describe how the graphs look the same, or how the graphs look different.

1. They are different shape because in the dice experiment it goes up and down in the marble experiment it goes straight up
2. The six sided dice looks like it gives the highest average score but they are the same.

Commentary

This response receives a score of 2 for ***mathematical content*** *because the student*

- completes both tally charts
- produces a single frequency graph that relates to the total data produced from both experiments. For example the frequency for a score of 5 is recorded as 23, rather than as 9 for the 12-sided die and 14 for the pair of 6-sided dice
- draws the contrasts from the shapes of the graph in Task 1 "in the dice experiment it goes up and down in the marble experiment it goes straight up" and Task 3, rather than comparing two frequency graphs in Task 3

This response receives a score of 2 for ***communication skills*** *because the student*

- completes the tally charts in a reasonably easy-to-read format
- completes the frequency graphs, with axes labeled appropriately, and frequency on the vertical axis
- leaves some gaps between the conclusions and the data tables, leaving the reader to fill in the relevant connections. There is no penalty for omitting the addition of the two frequencies. This is a significant omission, but this error has already been penalized heavily in the mathematical content score

This response would receive a score of	**1**	*for mathematical content*
and would receive a score of	**0**	*for communication skills*

Scoring Criteria

Mathematical Content

- A significant start made to the problem, such as (but not limited to) the completion of one observation chart, or the identification of crucial facts, like the rolling of a 1 only being possible for the 12-sided dice, or the conversion of any form of observation chart into a frequency graph.

Communication Skills

- insufficient evidence of communication skills.

Task 3: Does One 12-Sided Die Equal Two 6-Sided Dice?

Procedure

Throw the 12-sided die 50 times, and record the results in the third column of the first tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Throw the pair of 6-sided dice 50 times, and record the results in the third column of the second tally chart below. Combine these results with the results of the 50 throws that have already been entered in the second column of the tally chart. Record the total frequency in the fourth column of the tally chart.

Draw a frequency graph or histogram from **each** one of the tally charts.

From your tally charts and your graphs, answer the following question:

- Are the shapes of the graphs the same or different for the two experiments?

Tally Charts

Chart 1: 12-sided die:

Score (12-sided die)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	3	𝍸	4
2	5	|	1
3	4	𝍸 |	5
4	4	𝍸 𝍸	6
5	6	𝍸 𝍸 𝍸 |	11
6	3	||	2
7	5	𝍸	4
8	3	||	2
9	2	𝍸	4
10	6	||	2
11	5	𝍸 |	4
12	4	𝍸	5

Continued

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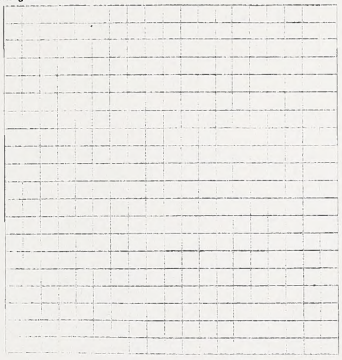
Chart 2: Pair of 6-sided dice:

Score (6-sided dice)	Frequency (1st 50)	Tallies for second 50	Total frequency
1	0		
2	2		
3	2		
4	3		
5	6		
6	7		
7	9		
8	7		
9	5		
10	5		
11	3		
12	1		

Frequency Graphs or Histograms

Note: there is a second grid on the next page if you want to draw each graph on its own grid.



Conclusions

Compare and contrast the shapes of the two graphs. Use one or two sentences to describe how the graphs look the same, or how the graphs look different.

Commentary

This response receives a score of 1 for ***mathematical content*** *because the student*

- produces tally charts for the second group of 50 tosses of the 12-side die, and just transfers the check marks in the third column to numbers in the fourth column of each tally chart. The tallies in the third column should have been added to the frequencies in the second column to produce larger totals
- does not attempt to produce the second tally chart or produce any frequency graphs

This response receives a score of 0 for ***communication skills*** *because the student*

- only accomplishes a preliminary task, the tally chart; since the major task was the transferring of frequency data into graphical form, there is insufficient evidence to assign a score for communication skills

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